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Electron dynamics in a strong laser field

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ABSTRACT

The three step model of high harmonic generation was used to investigate the electron behaviour in a strong laser field. The electron propagation was analyzed classically and the electron recombination with the parent atom was analyzed quantum mechanically. Three laser fields Argon with 500 nm, Helium-Neon with 632.8 nm and Ti-Sapphire with 800 nm wavelengths were used to analyze the electron behaviour for different intensities and the electron propagation was analyzed in cosine and sine electric laser fields. For a strong applied laser field, the atomic potential get distorted and the barrier potential is reduced. The maximum kinetic energy of the emitted electron, the ponderomotive energy was $8U_p$ in both cases and there were three points which gave the maximum kinetic energy for the laser field with sine electric field. When the laser intensity was increasing, the electron has a higher probability to move away from the parent atom. When the laser field changed its direction, electron recombined with the parent atom with a maximum kinetic energy of $3.17U_p$.

Keywords**:** Strong laser field; high harmonic generation; electron propagation; ponderomotive energy; Argon; Helium-Neon; Ti-Sapphire lasers

1. INTRODUCTION

The investigation on electrons in strong laser fields started with atoms is now carried out with high power short pulse lasers due to the rapid development of the technology. Electrons are responsible for the creation of the potential energy landscapes that drive atomic motion and adapt to faster timescales. The development of laser systems capable of reaching femtosecond pulse durations, helped to study more about intra molecular processes, chemical bond breaking and bond formation.

When a high power laser is focused into a gas of atoms, the atomic potential gets distorted reducing the potential barrier and the electrons have a possibility to tunnel through the reduced barrier [1]. Three possibilities occur with the reduction of the potential barrier. The first phenomenon recognized in 1979 is that the ground state electrons absorbed many photons which are very much higher than the threshold ionization called the above threshold ionization. Due to this process, the electrons acquire a higher kinetic energy. The second possibility recognized in 1975, is that one or more electrons emitted as a sequential process, called the direct or non-sequential ionization. In 1987, a third phenomenon, that the photons are emitted in the extreme ultraviolet (XUV) range, having harmonic behaviour was recognized [2]. The high harmonic generation discussed using these three step, laser ionization, propagation and recombination, was discovered by Corkum in 1993 [1]. High harmonic generation occurs when an intense laser beam directed into a gas or solid, the electric field of the laser beam is comparable with that of the atoms. The laser ionization and the recombination are quantum mechanical processes while the electron propagation can be treated as a classical process. The high harmonic generation capability of producing extreme-ultraviolet light opens the new field of ultrafast x-ray spectroscopy. The three step model of high harmonic generation led to the path of discovery of attosecond laser pulses [1].

In 2001 two teams independently demonstrated that high harmonic generation led to the emission of ultra-short light bursts, lasting just a few hundred attoseconds. These results sparked the emergence of the new field of attosecond science, which is now rapidly gaining ground worldwide [3]. When an electron was ejected from the parent atom, as the laser changes the direction, the electron accelerated back and re-collided with the parent ion [1]. A laser with a cosine electric field used to find the position of the electron classically showed that the electron recombined with the parent atom if the phase of ionization was less than 90º [4]. Moreover, a laser with a sine electric field was also used to find the position of the electron. When the electron re-collides with the parent iron, three possibilities can occur. The electrons scatter elastically or scatter inelastically or recombined with the parent iron emitting energy of the photons. Harmonics can be seen only in the third possibility and the produced high order harmonics propagated collinearly with the laser field. In the harmonic spectrum only the odd multiples were seen and there were three regions, perturbative, plateau and cut-off that can be identified with the characteristics of the spectrum [2].

In this work theoretical investigation of electrons in a strong laser field was carried out mainly focusing on the laser electric field's function. Laser electric field with the cosine function and the sine function were used for the simulations. The three step model of high harmonic generation was studied with the semi classical model. The second step of the three step model, the electron propagation was studied using the cosine and sine laser electric field. The positions of the electrons for three lasers with different wavelengths, Argon (Ar) 500 nm, Helium-Neon (He-Ne) 632.8 nm and Ti-Sapphire, 800 nm for intensities $1.6\times10^{18}\,\mathrm{Wm^{-2}}$, $5\times10^{18}\,\mathrm{Wm^{-2}}$ and $40\times10^{18}\,\mathrm{Wm^{-2}}$ are presented. The kinetic energy of the electron after tunnelling through the potential barrier was plotted with different laser wavelengths and intensities for the two cases of the laser electric fields. The kinetic energy of the recombined electron was studied using the Argon laser for $5\times10^{18}\,\mathrm{Wm^{-2}}$ and $40\times10^{18}\,\mathrm{Wm^{-2}}$ intensities.

2. LASER FIELD WITH COSINE ELECTRIC FIELD

A laser electric field, $E(t)$ linearly polarized in the x direction, is given by,

$$E(t) = E_0 \cos(\omega_0 t) \qquad (1)$$

where E_0 is the electric field amplitude and ω_0 is the electric field frequency. The force $F(t)$ that the electron of charge e experienced due to the laser electric field can be written from Newton's second law as

$$\int m dv = \int -e E_0 \cos(\omega_0 t) dt \qquad (2)$$

Integrating the above equation and using the initial condition that the electron is emitted at time t_i and the initial velocity $v(t_i)$ is zero, the velocity of the electron is,

$$v(t) = \frac{dx}{dt} = \frac{-eE_0}{m\omega_0}\left(\sin(\omega_0 t) - \sin(\omega_0 t_i)\right) \qquad (3)$$

Integrating both sides with respect to x and t respectively and with the initial condition that the initial displacement of the emitted electron $x(t_i)$ is at zero:

$$x(t) = \frac{eE_0}{m\omega_0^2}\left\{\cos(\omega_0 t) - \cos(\omega_0 t_i) + (\omega_0 t - \omega_0 t_i)\sin(\omega_0 t_i)\right\} \qquad (4)$$

Introducing the phase $\theta = \omega_0 t$, the position of the electron $x(t)$ after tunnelling through the reduced barrier can be rewritten as,

$$x(\theta) = \frac{eE_0}{m\omega_0^2}\left[\cos\theta - \cos\theta_i + (\theta - \theta_i)\sin\theta_i\right] \qquad (5)$$

The kinetic energy of the emitted electron can be found by using the classical equation of the kinetic energy,

$$E_k = \frac{m}{2}\left(-\frac{eE_0}{m\omega_0}\right)^2 (\sin\theta - \sin\theta_i)^2 \qquad (6)$$

Electron moves back and forth due to the oscillating electric field. If the electron pushes hard enough and the frequency is low enough, then the electron on average winds up with a lot of kinetic energy. This energy is called the U_p ponderomotive energy and is also known as the quiver energy:

$$U_p = \frac{e^2 E_0^2}{4m\omega_0^2} \qquad (7)$$

Equation for the kinetic energy can be rewritten in terms of ponderomotive energy U_p as,

$$E_k = 2U_p(\sin\theta - \sin\theta_i)^2 \qquad (8)$$

Electron propagation is purely a classical mechanical process and the maximum kinetic energy that the electron can acquire which produces high harmonic generation is $3.17U_p$. The cut off law states with the maximum harmonic photon energy E_c will be given by ionization potential I_p and the harmonic frequency f by

$$E_c = hf = I_p + 3.17U_p \qquad (9)$$

where h is the plank constant [4]. The position of the electron was analyzed using lasers with different wavelengths, Ar laser with 500 nm, He-Ne laser with 632.8 nm and Ti-Sapphire laser with 800 nm with different intensities for each laser. These are shown in Figure 1.

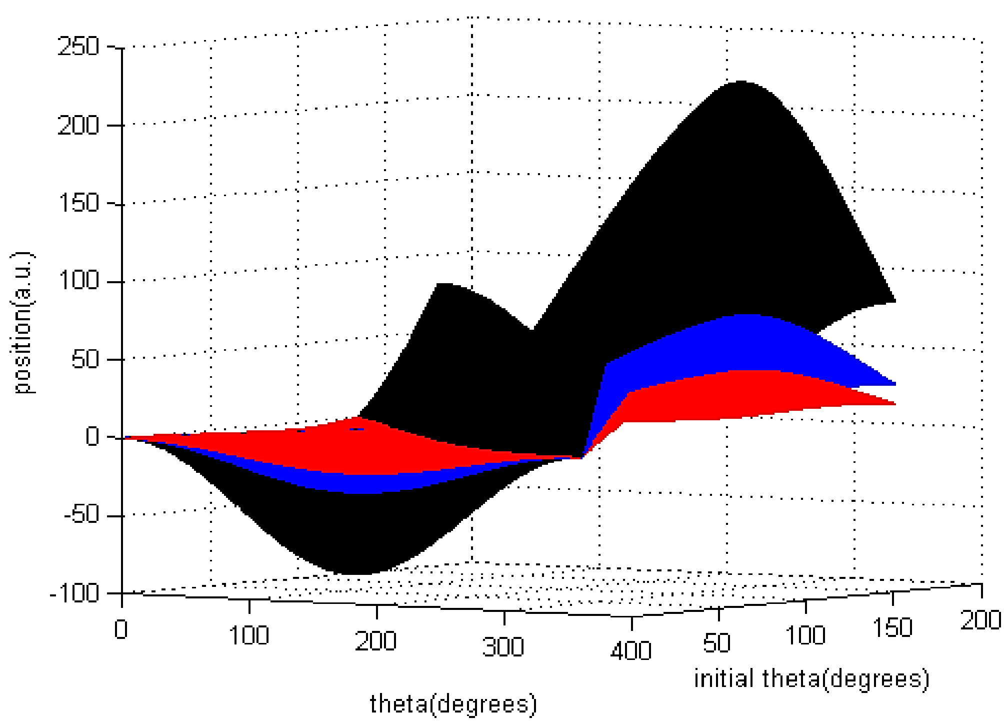


Figure 1(a). The position of the electron with different intensities for Ar laser. The red, blue and black colours correspond to the laser intensities of $1.6\times10^{18}\,\mathrm{Wm}^{-2}$, $5\times10^{18}\,\mathrm{Wm}^{-2}$ and $40\times10^{18}\,\mathrm{Wm}^{-2}$ respectively.

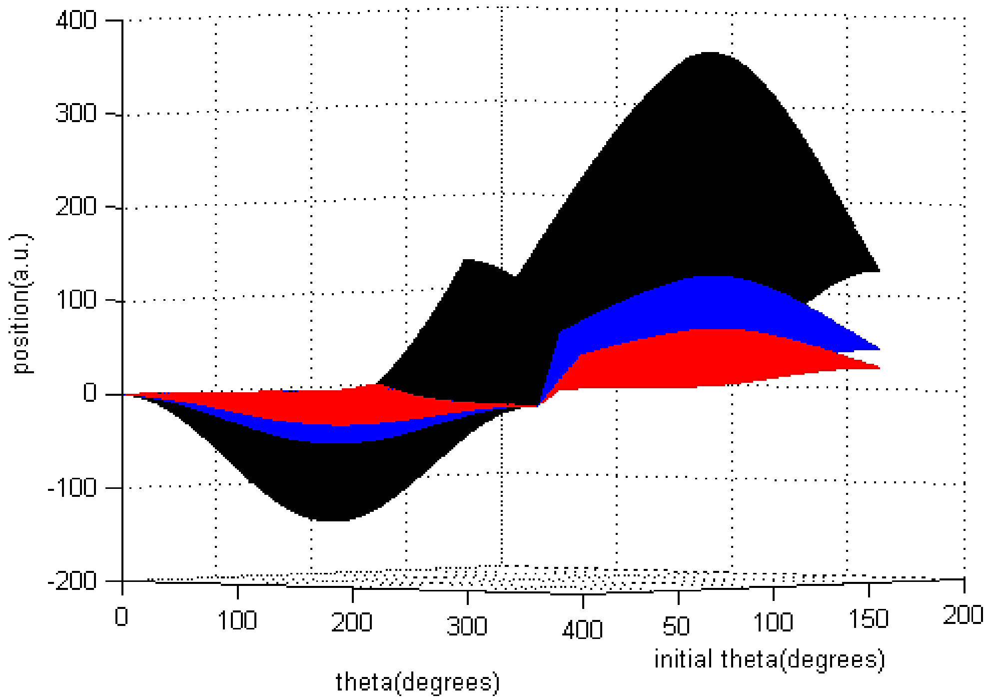


Figure 1(b). He-Ne laser

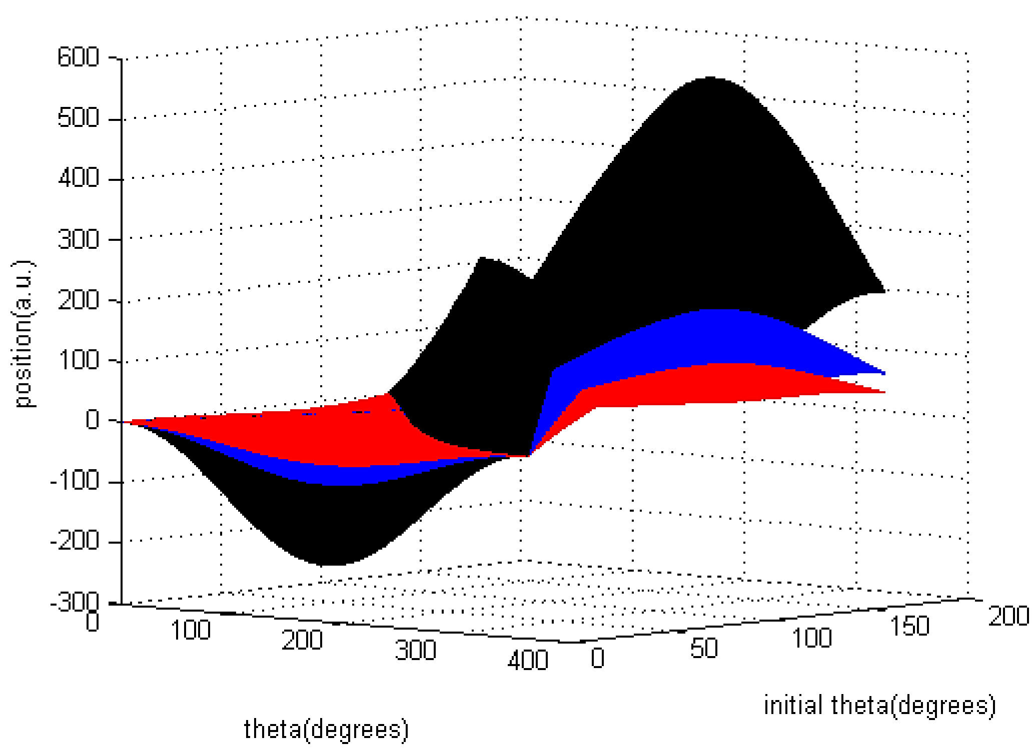


Figure 1(b,c). The position of the electron with different intensities for (*b*) He-Ne laser and (c) Ti-Sapphire lasers. The red, blue and black colour correspond to the laser intensities of $1.6\times10^{18}\,\mathrm{Wm^{-2}}$, $5\times10^{18}\,\mathrm{Wm^{-2}}$ and $40\times10^{18}\,\mathrm{Wm^{-2}}$ respectively.

The data points with red, blue and black correspond to the laser intensities of $1.6\times10^{18}\,\mathrm{Wm}^{-2}$, $5\times10^{18}\,\mathrm{Wm}^{-2}$ and $40\times10^{18}\,\mathrm{Wm}^{-2}$ respectively. The position of the electron is measured in atomic units and all the angles are in degrees.

When the wavelength and intensity of the laser is increasing, the position of the electron from the centre of the atom increases. For all the three lasers, the maximum position of the electron for different intensities along the positive direction of the x-axis occurs at a same point with the same coordinates.

Similarly, the maximum position of the electron for different intensities along the negative direction of the x-axis occurs at the same point and that point has the same coordinates for the three lasers.

The distribution of the position of the electron is not symmetric and the maximum distance of the electron for the intensity $40\times10^{18}\,\mathrm{Wm}^{-2}$ along the positive x-axis are 231.451, 370.726 and 592.516 atomic units and along the negative axis that value is -81.080, -129.869 and -207.565 atomic units respectively for Ar, He-Ne and Ti-Sapphire laser. Furthermore, it can be seen that the three planes are overlapping at some point and electron will recombine with the parent atom when the position of the electron is zero.

The maximum position of the electron for different intensities along the positive direction of the x-axis occurs at a same point with a phase angle of 359.817^{o} and an initial angle of 89.954^{o}. Similarly, the maximum position of the electron for different intensities along the negative direction of the x-axis occurs at the same point with a phase angle of 179.908^{o} and an initial angle of 0°.

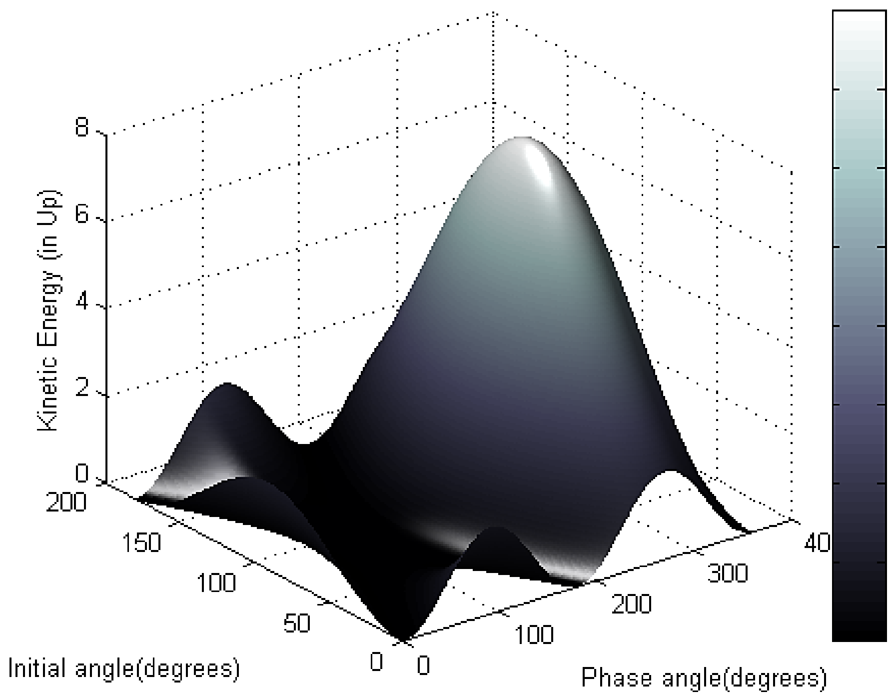
The three dimensional plot of the kinetic energy variation with the phase angles and the initial angles are shown in Figure 2 for the direct and side (x-z) views. Initial angle varies from 0º to 180º and it is also known as the phase of ionization. The phase angle varies from 0º to 360º. The graph of kinetic energy of the recombined electron variation with the phase angle and the initial angle is shown in Figure 3.

All the measurements were in atomic units. The maximum kinetic energy that the electron can acquire goes up to a maximum of $8U_P$ at the initial angle of 89.954º and the phase angle of 269.863^{o}. The electron which tries to recombine with the parent atom with the maximum kinetic energy of $3.17U_P$ has a phase angle between, 195° and 345°. The angle of recombination variation with the phase of ionization using Ar laser for the intensities $5\times10^{18}\,\mathrm{Wm}^{-2}$ and $40\times10^{18}\,\mathrm{Wm}^{-2}$ are shown in figure 3. The initial angle of the electron should be less than 90º in order to recombine with the parent atom. Some of the initial angles have two angles of recombination.

Also most of the time, the initial angle and angle of recombination represented by the solid blue linear line are equal. As the intensity is increased only a few data points are present, indicating that the probability of re-collision of the electron with the parent atom is lower. Furthermore, the laser intensity is an effective factor of electron's displacement.

The graph of kinetic energy of the recombined electron variation with the phase angle and the initial angle using Ar laser for the intensities $5\times10^{18}\,\mathrm{Wm}^{-2}$ and $40\times10^{18}\,\mathrm{Wm}^{-2}$ are shown in Figure 4.

The kinetic energy of the emitted electron depends on the phase of ionization and the angle of recombination that is phase angle.



2(a)

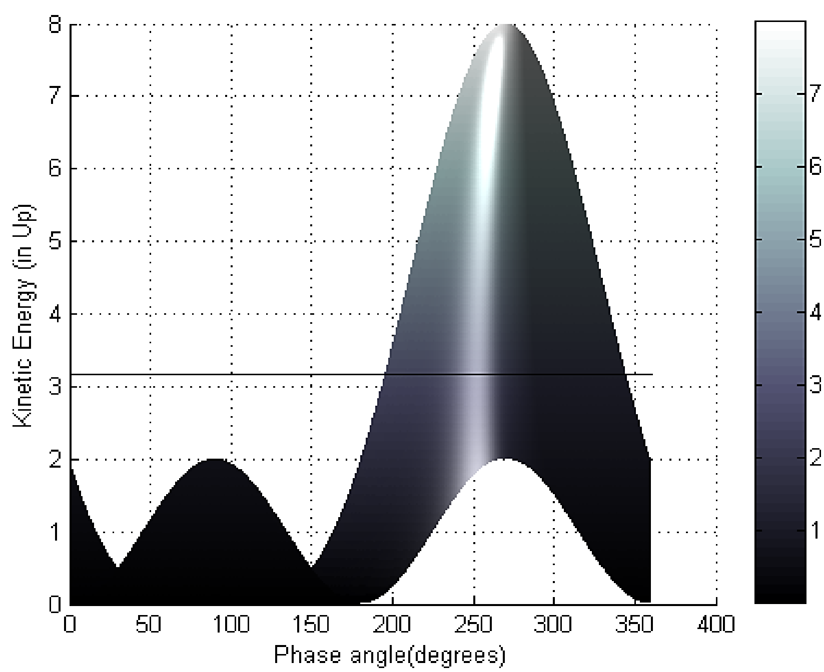
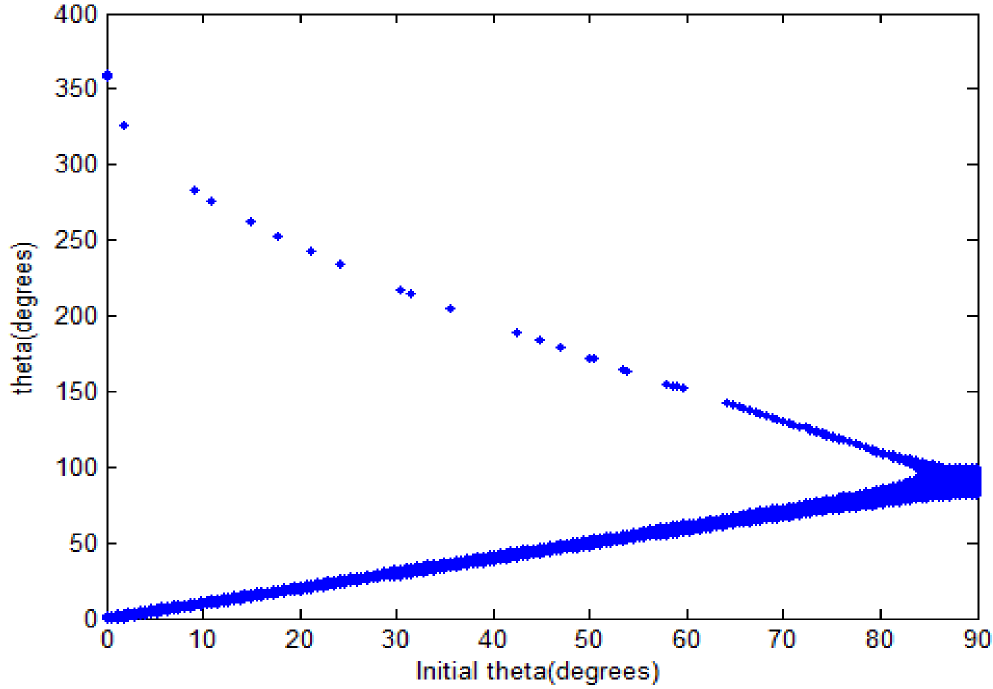


Figure 2. Electron kinetic energy variation due to a cosine laser electric field (*a*) 3D view and (*b*) side view (x-z axis)



3(a)

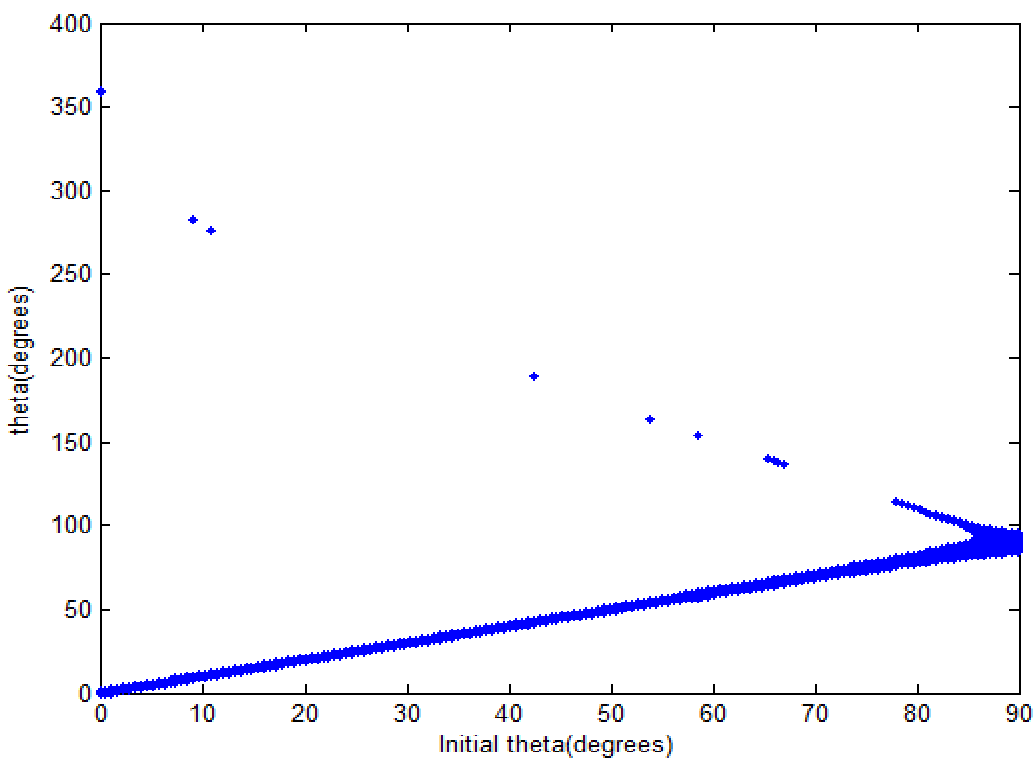
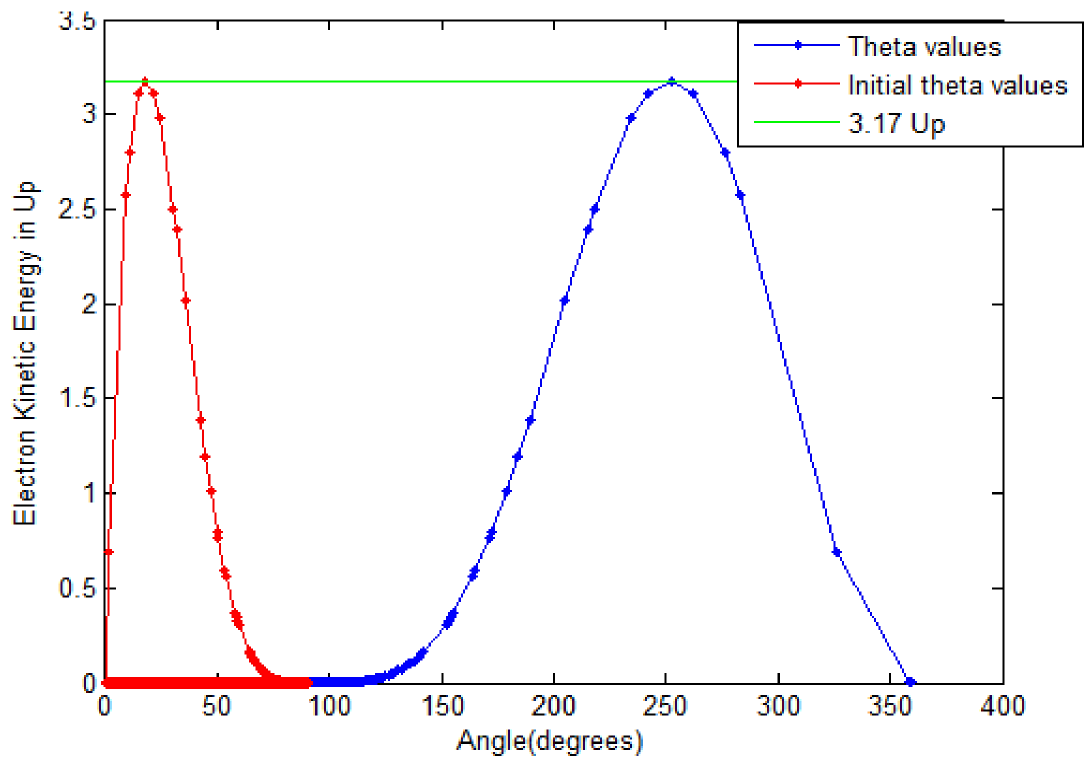


Figure 3. Angle of recombination variation with the phase of ionization for Ar laser (500 nm) with cosine electric field for intensities (*a*) 5×10^{18} Wm^{-2} and (*b*) 40×10^{18} Wm^{-2} .



4(a)

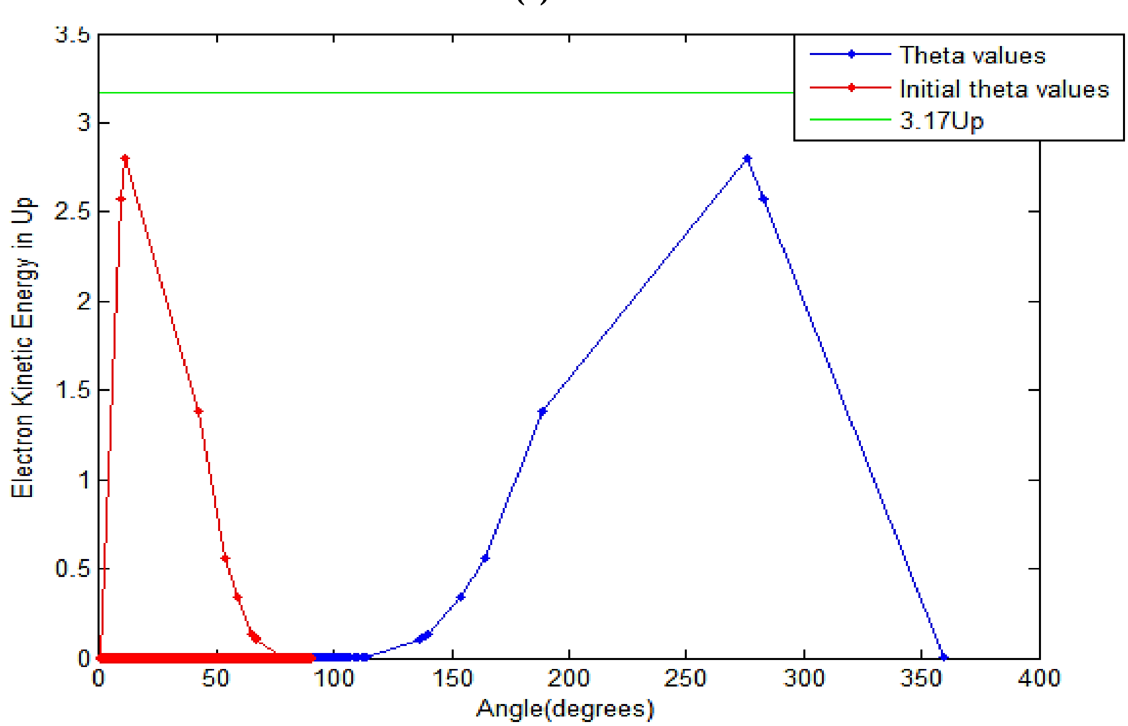


Figure 4. The kinetic energy variation of the recombined electron for the cosine electric field with initial angle and phase angle for the Ar laser (500 nm) with intensities (*a*) 5 × 10^{18} Wm^{-2} and (*b*) 40 × 10^{18} Wm^{-2}

According to the graph, red and blue colour data points represent the kinetic energy variation due to initial angle and phase angle respectively. For laser intensity $5\times10^{18}\,Wm^{-2}$ the maximum kinetic energy of 3.170U_p and its corresponding initial angle and phase angle are 17.7º and 252.7º respectively while for $40\times10^{18}\,Wm^{-2}$ intensity, the maximum kinetic energy is 2.80U_p and the corresponding initial angle and phase angle are 10.9º and 276.2º respectively. Though the maximum kinetic energy that the recombined electron acquires changes with the laser intensity, this value however will not exceed the value of 3.17 U_p.

4. LASER FIELD WITH A SINE ELECTRIC FIELD

The laser sine electric field $E(t)$, linearly polarized in the x direction, is given by,

$$E(t) = E_0 \sin(\omega_0 t) \qquad (10)$$

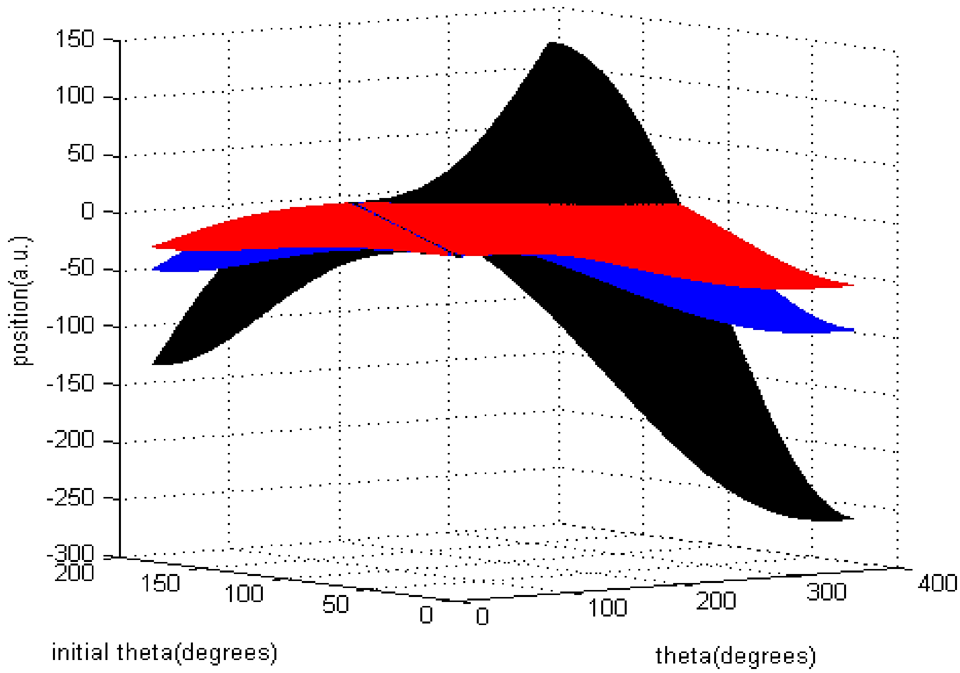
Using the same procedure as in the cosine electric field, the position of the electron after tunnelling through the reduced barrier can be written as,

$$x(\theta) = \frac{eE_0}{m\omega_0^2}\left[\sin\theta - \sin\theta_i + (\theta_i - \theta)\cos\theta_i\right] \qquad (11)$$

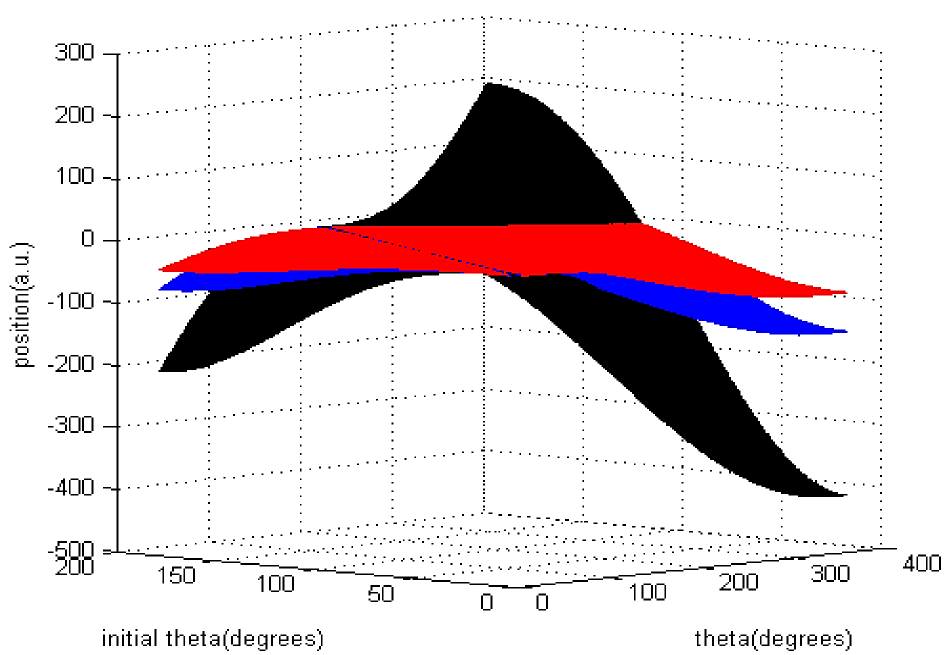
and the kinetic energy of the electron

$$E(\theta) = 2U_p\left(\cos\theta - \cos\theta_i\right)^2 \qquad (12)$$

The position of the electron obtained using, Ar, He-Ne and Ti-Sapphire lasers with different intensities are shown in Figure 5. For each laser the data points presented in red, blue and black correspond to the laser intensities of $1.6\times10^{18}\,Wm^{-2}$, $5\times10^{18}\,Wm^{-2}$ and $40\times10^{18}\,Wm^{-2}$ respectively. The position of the electron is measured in atomic units and all the angles are in degrees. As the wavelength of the laser increases, the position of the electron's maximum distance from the parent atom also increases. For laser intensity of 40×10^{18} Wm^{-2}, the maximum distances of the electron with respect to Ar, He-Ne and Ti-Sapphire lasers along the positive x-axis are 127.102, 203.585 and 325.382 atomic units and along the negative axis -254.721, -407.998 and -652.087 atomic units respectively. With different laser electric fields, the maximum position of the electron from the parent atom changes. For three different lasers, there is a higher probability that the electron is at the negative direction of the x axis. For all the three lasers, the maximum position of the electron for different intensities along the positive direction of the x axis occurs at a same point and that point has the same coordinates for the three lasers. Similarly, the maximum position of the electron for different intensities along the negative direction of the x axis occurs at a same point and that point has the same coordinates for the three lasers as well.

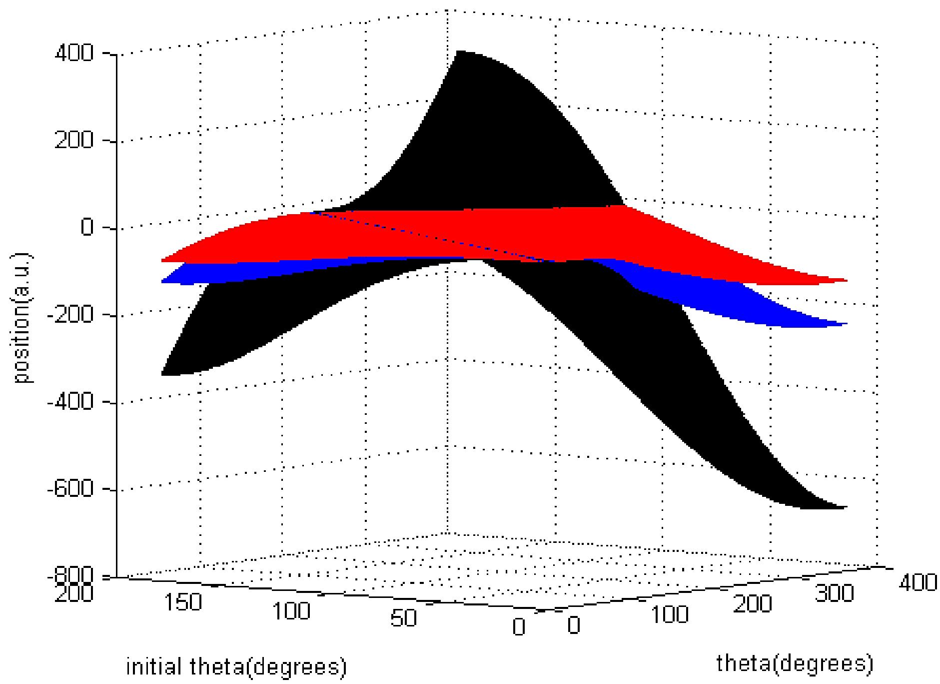


5(a): Ar laser



5(b): He-Ne laser

Figure 5. The position of the electron with different intensities (*a*) Ar laser, (*b*) He-Ne laser for sine electric field. The red, blue and black colour correspond to the laser intensities of $1.6\times10^{18}\,\mathrm{Wm^{-2}}$, $5\times10^{18}\,\mathrm{Wm^{-2}}$ and $40\times10^{18}\,\mathrm{Wm^{-2}}$ respectively.



5(c): Ti-Sapphire laser

Figure 5(c). The position of the electron with different intensities for Ti-Sapphire laser for sine electric field. The red, blue and black colour correspond to the laser intensities of $1.6\times10^{18}\,\mathrm{Wm}^{-2}$, $5\times10^{18}\,\mathrm{Wm}^{-2}$ and $40\times10^{18}\,\mathrm{Wm}^{-2}$.

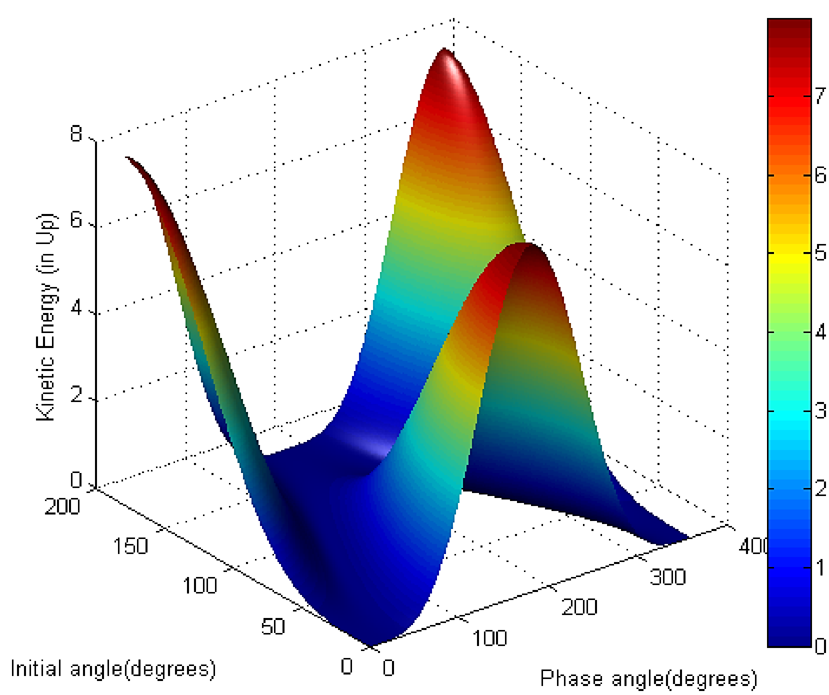


Figure 6(a). Electron Kinetic Energy variation due to a sine laser electric field (*a*) 3D view

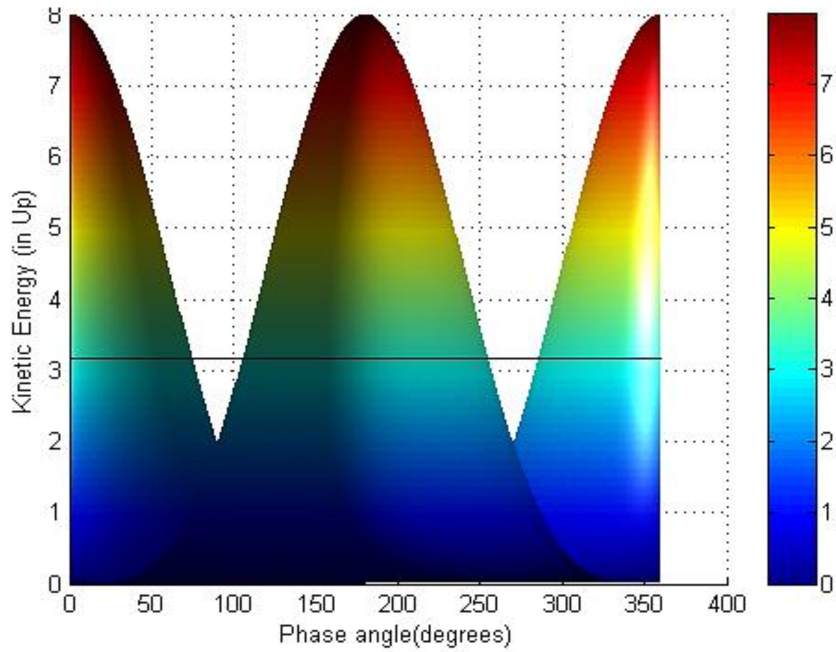


Figure 6(b). The side view (*x*-*z* axis) of electron kinetic energy variation due to a sine laser electric field.

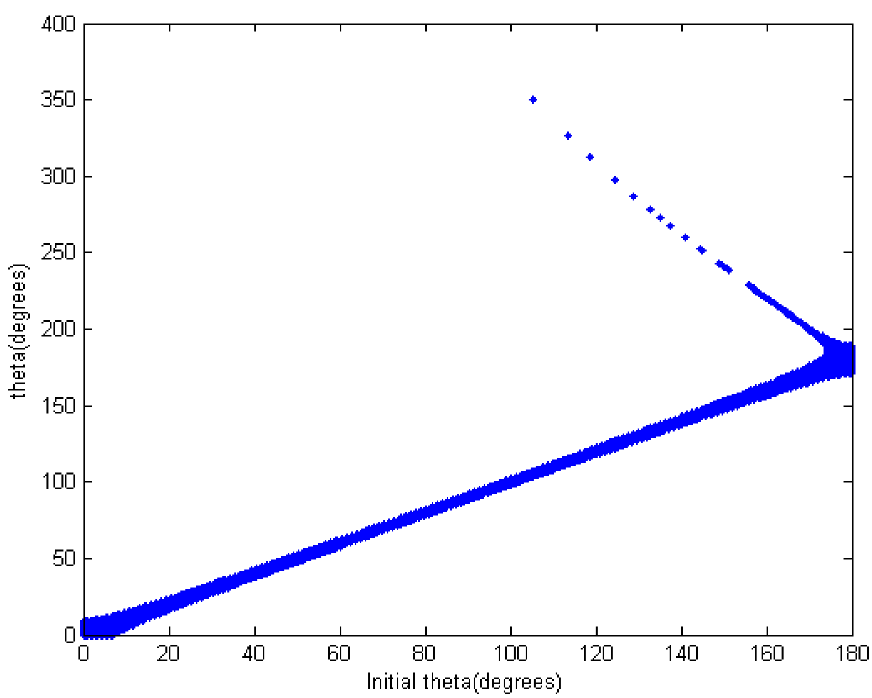


Figure 7(a). Angle of recombination variation with the phase of ionization for the Ar laser with a sine electric field intensity of 5×10^{18} Wm^{-2}

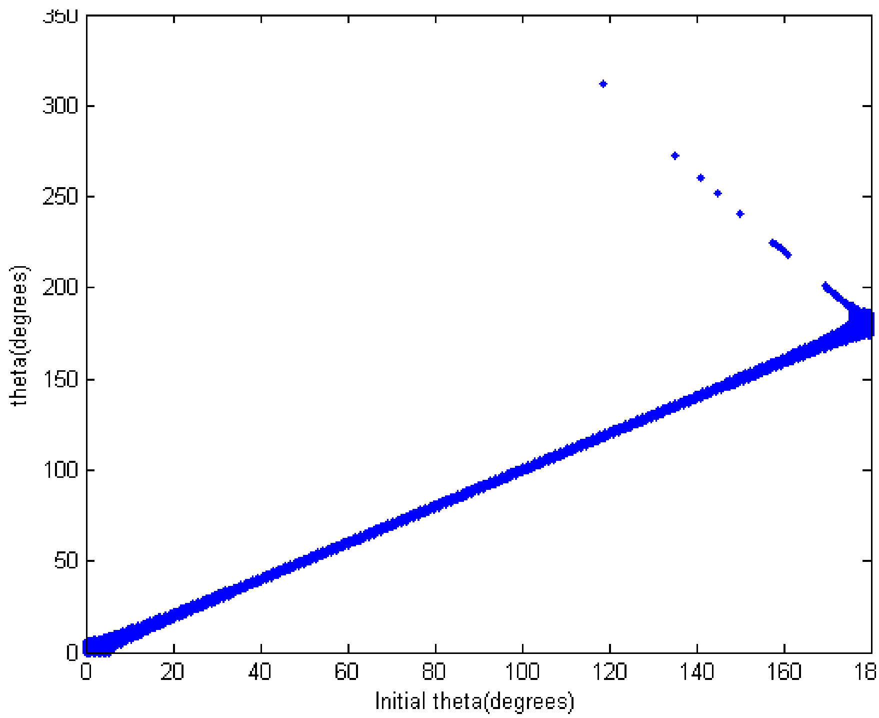


Figure 7(b). Angle of recombination variation with the phase of ionization for the Ar laser with a sine electric field intensity of 40×10^{18} Wm^{-2}

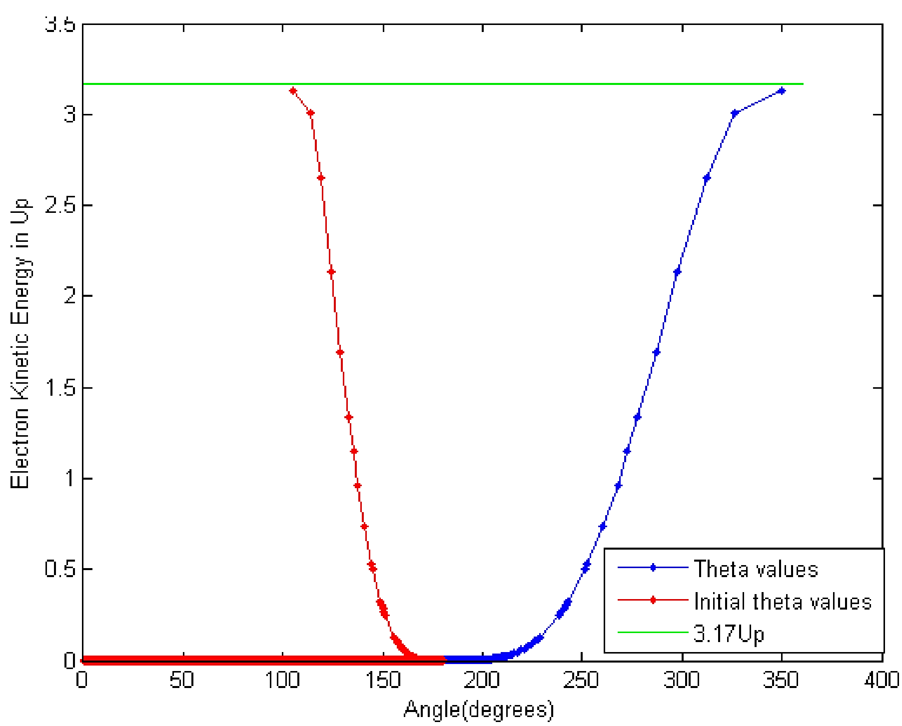


Figure 8(a). Kinetic energy variation of the recombined electron with initial angle and phase angle with Ar laser with intensity 5×10^{18} Wm^{-2} for the sine electric field.

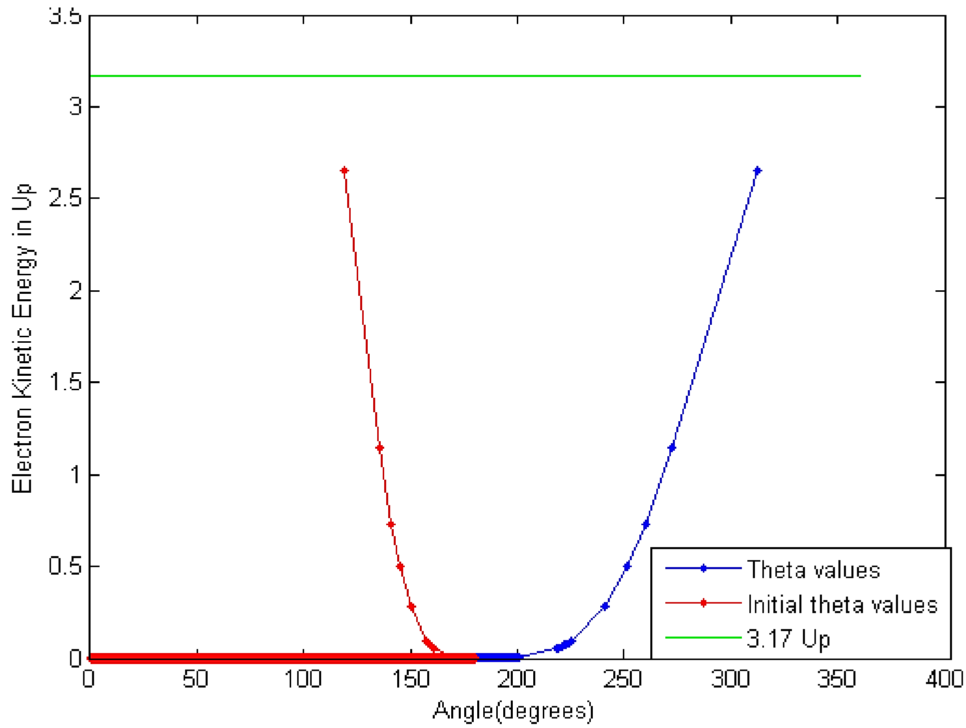


Figure 8(b). Kinetic energy variation of the recombined electron with initial angle and phase angle with Ar laser with intensity 40 × 10^{18} Wm^{-2} (sine electric field).

The kinetic energy of the emitted electron was plotted by considering the initial angle and the phase angle. Initial angle known as the phase of ionization varies from 0° to 180° while the phase angle varies from 0° to 360°. The maximum kinetic energy of the electron goes up to a maximum of $8U_{p,}$ but the electron which comes back and re-collides with the parent atom has a maximum of $3.17U_p$. According to the graph, there are three points which give the maximum kinetic energy of $8U_p$ with phase angles 0°, 180° and 360° corresponding to initial angles of 180°, 0° and 180° respectively.

The angle of recombination variation with the initial angle which gave the zero displacement of the electron for the sine electric field Ar laser with 5 × 10^{18} Wm^{-2} and 40 × 10^{18} Wm^{-2} intensities are presented in Figure 7. For the electron recombination with the parent atom where the phase of ionization, that is the initial angle varies from zero to 180 degrees. Compared to the graph which was drawn for the Ar laser with cosine electric field with the same intensity, there are fewer points for the sine electric field having a phase angle over 180°. As the intensity of the Ar laser was increased, the probability that the electron will recombine with the parent atom decreases.

The kinetic energy variation of the recombined electron with initial angle and phase angle for the Ar laser with intensities 5 × 10^{18} Wm^{-2} and 40 × 10^{18} Wm^{-2} are shown in Figure 8. The maximum kinetic energy of the recombined electron with sine electric field with an intensity of 40 × 10^{18} Wm^{-2} is $2.651U_p$. The corresponding initial angle and phase angle are 118.6° ±0.1° and 312.3° ±0.1° respectively. As the laser intensity increases, the maximum kinetic energy that the recombined electron acquires decreases. However this value will not exceed $3.17U_p$.

5. CONCLUSIONS

When an external laser field is applied to the atomic potential, it gets distorted. The atomic potential distortion depends on the laser intensity and wavelength. With higher laser intensity and wavelength, the electron tunnelling probability increases. The position of the electron for the cosine laser electric field and for the sine laser electric field showed that the laser field intensity and the laser field wavelength have a higher effect on the position of the electron. The kinetic energy distributions for the two laser electric fields are totally different. However, the maximum kinetic energy that the emitted electron can acquire is $8U_p$. There are three points which give a maximum kinetic energy of $8U_p$ for the sine laser electric field and on the other hand there is only a single point of maximum kinetic energy of $8U_p$ for the cosine electric field. When the electron recombines with the parent atom with the maximum kinetic energy of the recombined electron is $3.17U_p$. Furthermore, the kinetic energy of the recombined electron will change with the laser field intensity without exceeding the maximum value. The initial angle which can be identified as the phase of ionization has to be less than 90° in order to recombine with the parent atom if the laser electric field is a cosine field. Moreover, the initial angle should be between 90° – 180° in order to recombine with the parent atom, if the laser electric field is a sine field. When the laser intensity is increasing, the probability that the electron recombine with the parent atom will decrease.

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