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Flow of viscous incompressible MHD fluid over a suddenly accelerated plate with variable electroconductivity and chemical reaction provoked by radiation

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ABSTRACT

A study of a suddenly accelerated plate with variable approximated electronegativity with MHD fluid provoked by chemical reaction and radiation is made. The governing hydrodynamical equations of the physical problem was formulated and its solution and analysis showed that increase in electroconductivity, prandtl number, Reynold's number, Grashof number, chemical reaction and radiation all result to a corresponding increase in the velocity profile of the fluid while magnetic field increase results to a decrease in the velocity profile of the fluid. The shear stress at the wall of the plate was also determined.

***Keywords*:** MHD, Electrconductivity, Prandtl number, Grashof number, Reynold's Number, Chemical reaction, Radiation

1. INTRODUCTION

Magnetohydrodynamics (MHD) is primarily concerned with the interaction of electrically conducting fluids and electromagnetic fields. When a conducting fluid moves through a magnetic field, an electric field and consequently a current may be induced and in turn the current interacts with the magnetic field to produce a body force on the fluid [1]. MHD flow occurs in the sun, the earth interior, the ionosphere and many more. In the laboratory, many devices such as metallurgical processes, electron beam dynamics, travelling tubes and electrical discharges all utilize MHD interactions directly. Recently, MHD, thermal radiation and chemical reaction appear in various engineering processes, agriculture, plasma studies and petroleum industries.

The interplay of these parameters under consideration in the study of suddenly accelerated plate cannot be overemphasized. Various scholars have contributed to the study of MHD and its ancillary parameters, [2] examined MHD boundary layer flow past a wedge and made useful findings. [3] and [4] considered the influence of radiation on MHD Couette flow and Poiseuille flow in a porous medium and its analysis in part were in agreement with the study of [5].

A similar study was also carried out by [6], where the effect of radiation and chemical reaction on the depletion of the Ozone layer was investigated. Earlier, [7] and [8] studied the effect of radiation and other parameters on MHD flow of fluid and made useful findings while [9] examined the problem of MHD Couette flow with heat transfer between two horizontal plates in the presence of a uniform transverse magnetic field. [10], also tackled the problem of MHD Couette-Poiseuille flow in a porous medium and opined in part that Reynold's number increases as temperature profile increases and magnetic field increases as velocity profile decreases. Our aim is to extend the study of [5] by incorporating chemical reaction and radiation, this practice will complement the work of [2] and widens the applicability of studies of this nature.

Nomenclature

ρ = Fluid density
u' = fluid velocity
P = fluid pressure
μ = absolute viscosity
g = acceleration due to gravity
t = dimensionless time
T = fluid temperature
C = dimensionless fluid concentration
a = thermal diffusivity
q_z = radiative term
K_r^2 = chemical reaction term
y' = coordinate
σ = Stefan Boltzmann constant
C_p = specific heat at constant pressure
T_o = initial temperature
C_o = initial concentration
β_c = coefficient of volume expansion for concentration
β_T = coefficient of volume expansion for temperature
T_∞ = reserviour temperature
$\wedge$ = Planck's function

α_{K^*} = absorption coefficient
K^* = frequency of radiation
ρ_∞ = reserviour density
Θ = dimensionless temperature
Gr_c = free convection parameter due to concentration
Gr_T = free convection parameter due to temperature
k = dimensionless chemical reaction term
t' = time
y = dimensionless coordinate
υ = kinematic viscosity
C' = fluid concentration
R = dimensionless radiation term
Re = Reynolds number
u = dimensionless fluid velocity

2. MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

Let there be a flat plate as shown in figure 1, extending to large distances in the x and z-directions. Let there be an incompressible viscous fluid over a half plane y = 0. Let the fluid in contact with the plate be infinite in extent and let it be at rest at time t < 0. At t = 0, the plate is suddenly set in motion at a constant velocity U in the x-direction. This generates a two dimensional parallel flow near the plate as a result of the parameters affecting the motion of the fluid.

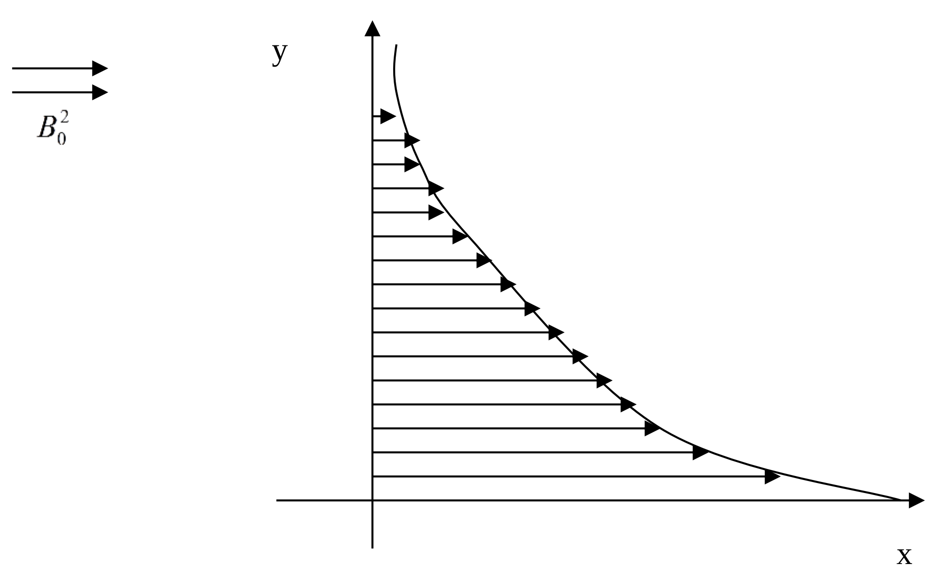

Figure 1. The physical model and coordinate system of the problem.

Since the plate is situated in an infinite fluid, the pressure must be constant every where and the governing equations is therefore

$$\frac{\partial u'}{\partial t'} = \mu \frac{\partial^2 u'}{\partial y'^2} + g\beta_T (T - T_\infty) + g\beta_c (C - C_\infty) - \sigma \frac{B_0^2 u'}{\rho} - \frac{\sigma_\infty u'}{U} \quad (1)$$

where following [11], the fluid electro conductivity is assumed to be of the form $\sigma_\infty \left(1 - \frac{u'}{U}\right)$ but for physical exigency and mathematical amenability, it is approximated to the form in (1)

$$\frac{\partial T}{\partial t'} = a \frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho C_p} \nabla . q_z \quad (2)$$

$$\frac{\partial C'}{\partial t'} = a \frac{\partial^2 C'}{\partial y'^2} - \frac{1}{\rho C_p} k_r^2 C' \quad (3)$$

$$\frac{\partial^2 q_z}{\partial y'^2} - 3\sigma^2 q_z - 16\sigma T_\infty^3 \frac{\partial T}{\partial y'} = 0 \quad (4)$$

For optically thin medium with relatively low density in the spirit of [12], equation (4) reduces to

$$\frac{\partial q_z}{\partial y'} = 4\delta^2 (T - T_\infty) \quad (5)$$

where $\delta^2 = \int_0^\infty (\alpha_{K^*} \frac{\partial \wedge}{\partial T}) dk^*$

subject to the boundary and initial conditions

$$u' = 0 \text{ when } t \le 0 \text{ for all } y' \quad (6)$$

$$u'(0) = U, \quad u'(\infty) = 0 \quad (7)$$

$$T(0) = 1, \quad T(\infty) = 0 \quad (8)$$

$$C'(0) = 1, \quad C'(\infty) = 0 \quad (9)$$

Dimensionless variables

For dimensional homogeneity of the governing fluid equations, we substitute the following dimensionless quantities or expressions of the fluid variables

$$u = \frac{u't}{y'}, M = \frac{\sigma B_0^2 y'^2}{\rho\mu\upsilon}, R = \frac{4\delta^2 \rho_\infty C_\infty y'^2}{\rho C_p \upsilon}, C = \frac{C' - C_o}{C' - C_\infty}, \sigma_0 = \frac{\sigma_\infty t'}{\sigma u' y'}$$

$$\Pr^{-1} = \frac{\upsilon}{a}, g' = \frac{gy'}{u'^2}, \theta = \frac{T - T_o}{T - T_\infty}, U' = \frac{U}{u}, \mathrm{Re}^{-1} = \frac{\mu}{u'y'\rho}$$

$$t = \frac{u'y'}{t'}, Gr_T = \frac{g\beta_T (T - T_o) y'^3}{u'^3}, Gr_c = \frac{g\beta_c (C' - C_o) y'^3}{u'^3}, k = \frac{\kappa_r^2 T_\infty \upsilon}{\alpha u'^2}$$

into equations (1) – (5), transformed them into

$$\frac{\partial u}{\partial t} = \mathrm{Re}^{-1} \frac{\partial^2 u}{\partial y^2} + Gr_T \theta + Gr_C C - Mu - \sigma_0 u \qquad (10)$$

$$\frac{\partial \theta}{\partial t} = \Pr^{-1} \frac{\partial^2 \theta}{\partial y^2} - R\theta \qquad (11)$$

$$\frac{\partial C}{\partial t} = \Pr^{-1} \frac{\partial^2 C}{\partial y^2} - kC \qquad (12)$$

subject to the boundary and initial conditions

$$u = 0 \text{ when } t \le 0 \text{ for all } y \qquad (13)$$

$$u(0) = U', u(\infty) = 0 \qquad (14)$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \qquad (15)$$

$$C(0) = 1, \quad C(\infty) = 0 \qquad (16)$$

3. METHOD OF SOLUTION

Following the transformation employed by [7] for sufficiently large values of the time variable, we seek solution of (10) – (12) of the forms

$$u = u_1(y) e^{-nt} \qquad (17)$$

$$\theta = \theta_1(y) e^{-nt} \qquad (18)$$

$$C = C_1(y) e^{-nt} \qquad (19)$$

where n is a constant.

The boundary conditions are also transformed into the forms

$$u_1(0) = U'e^{nt}, \quad u_1(\infty) = 0 \tag{20}$$

$$\theta_1(0) = e^{nt}, \quad \theta_1(\infty) = 0 \tag{21}$$

$$C_1(0) = e^{nt}, \quad C_1(\infty) = 0 \tag{22}$$

Equations (10) – (12) can be rewritten using equations (17) – (19) as

$$u_1''(y) - \mathrm{Re}(M - \sigma_o - n)u_1(y) = -\mathrm{Re}(Gr_T\theta_1(y) - Gr_C C_1(y)) \tag{23}$$

$$\theta_1''(y) - \Pr(R - n)\theta_1(y) = 0 \tag{24}$$

$$C_1''(y) - \Pr(k - n)C_1(y) = 0 \tag{25}$$

Solutions of equations (24) and (25) after imposing the boundary conditions (21) and (22) respectively as well as substituting into equations (18) and (19), we get

$$\theta = Cosha_1 y \tag{26}$$

$$C = Cosha_2 y \tag{27}$$

where $a_1 = (\Pr(R - n))^{0.5}$ and $a_2 = (\Pr(k - n))^{0.5}$

To solve equation (23), we substitute equations (26) and (27) into it and solve the resulting equation, this is followed by imposing the boundary conditions and substitute in equation (17) to get

$$u(y,t) = e^{-nt}\left[(Ue^{nt} - A_1 - B_1)Cosh\lambda y + A_1 Cosha_1 y + Cosha_2 y\right] \tag{28}$$

where $A_1 = -\dfrac{\mathrm{Re}\,Gr_T}{a_1^2 - \lambda}$, $B_1 = -\dfrac{\mathrm{Re}\,Gr_C}{a_2^2 - \lambda}$ $\lambda = \sqrt{\mathrm{Re}(M - \sigma_0 - n)}$

The shear stress at the plate is given by

$$\sigma_{yx} = \mu\frac{du}{dy} = \mu e^{-nt}\left[\lambda(Ue^{nt} - A_1 - B_1)Sinh\lambda y + A_1 a_1 Sinha_1 y + a_2 Sinha_2 y\right] \tag{29}$$

4. RESULTS AND DISCUSSION

In order to get physical insight and numerical validation of the problem, an approximate value of constant velocity $(U = 5.36)$ and constant $(\lambda = 0.0035)$ is chosen. The values of other parameters made use of are

$\mathrm{Re} = 10{,}00, 20.00, 30.00.40.00, 50.00$

$Gr = 1.82, 3.82.5.82, 7.82, 9.82$

$M = 2.33, 4.33, 6.33, 8.33, 10.33$

$\sigma_0 = 1.91, 2.91, 3.91, 4.91, 5.91$

$\mathrm{Pr} = 0.31, 0.41, 0.51, 0.61, 0.71$

$R = 2.50, 3.50, 4.50, 5.50, 6.50$

$k = 0.73, 1.03, 1.33, 1.63, 1.93$

$t = 1$

In the discussion, we start with the Reynold's number as shown in Figure 2. Increase in Reynold's number results in an increase in the velocity profile of the fluid and this result is consistent with an earlier study of [11]. Figure 3, shows that increase in prandtl number brings about a corresponding increase in the velocity profile of the fluid and the result is in agreement with an earlier study of [7]. Figure 4, displays increase in magnetic field and a corresponding decrease in velocity profile of the fluid.

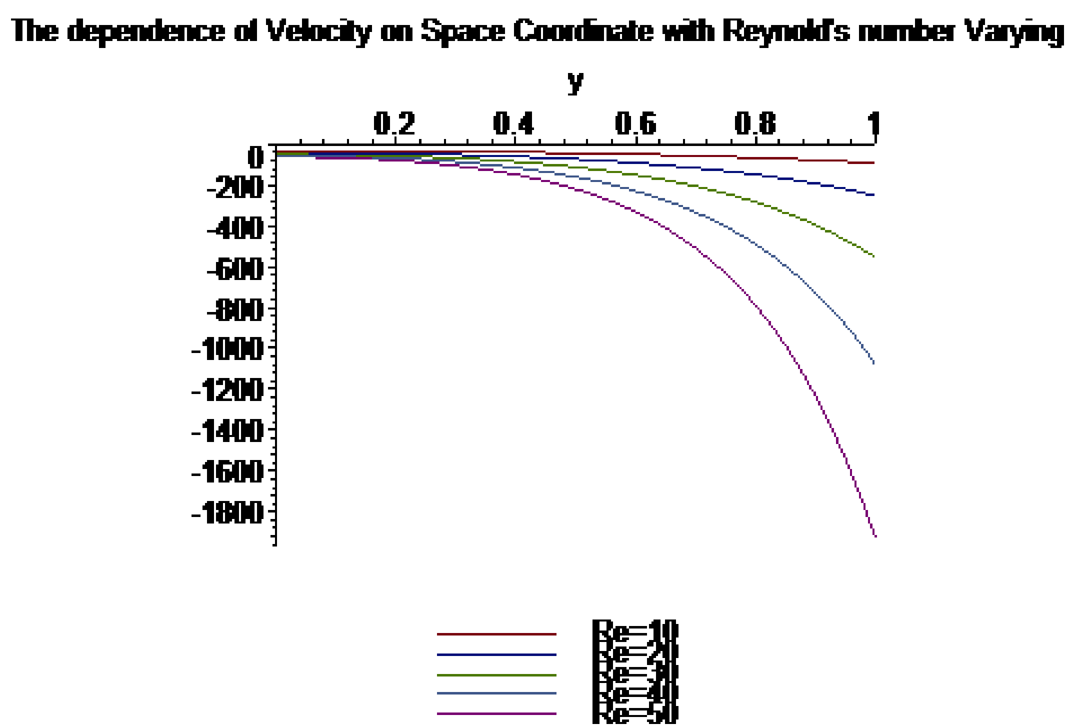

Figure 2. Velocity profile u against boundary layer y for varying Reynolds number Re.

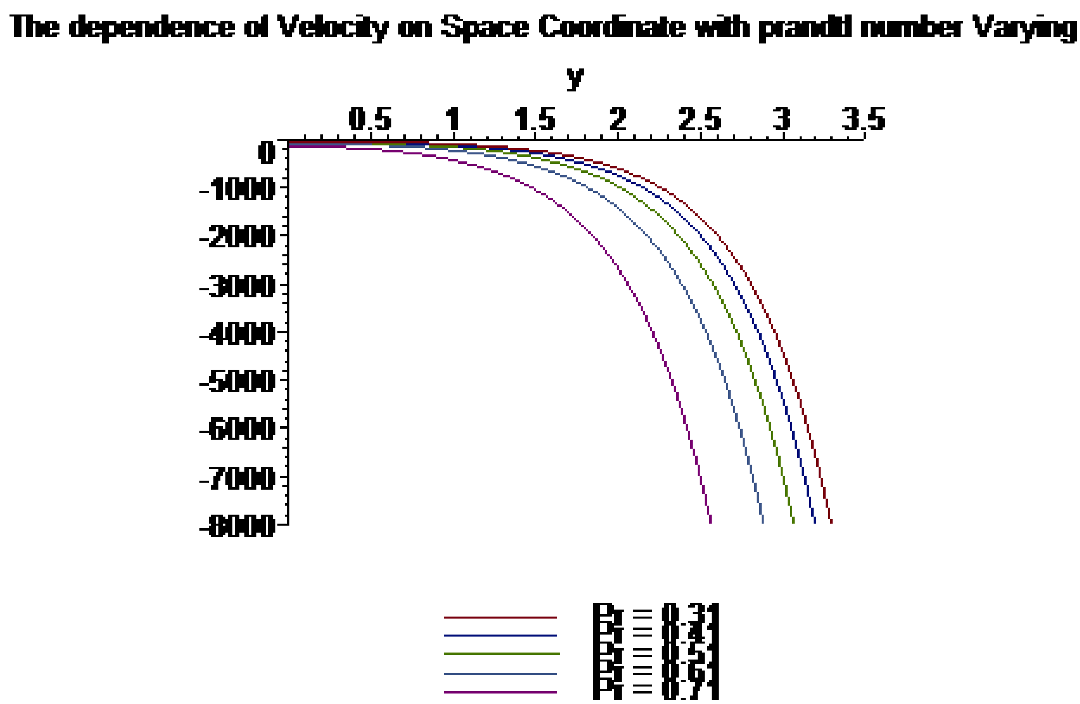

Figure 3. Velocity profile u against boundary layer y for varying Prandtl number Pr

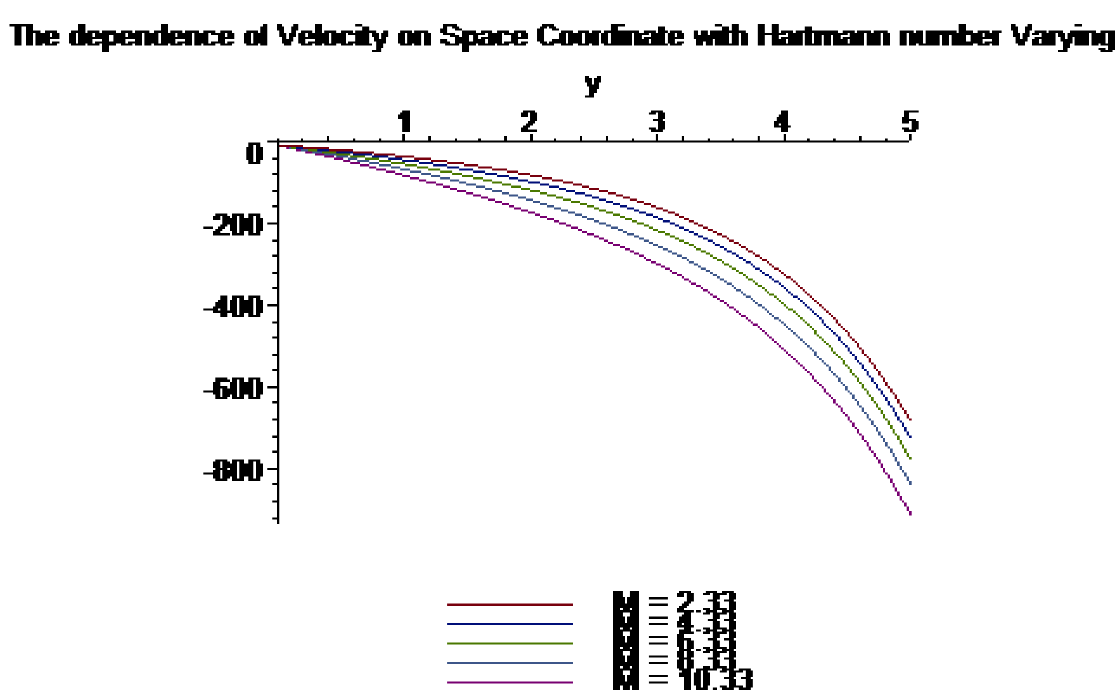

Figure 4. Velocity profile u against boundary layer y for varying Hartmann number M

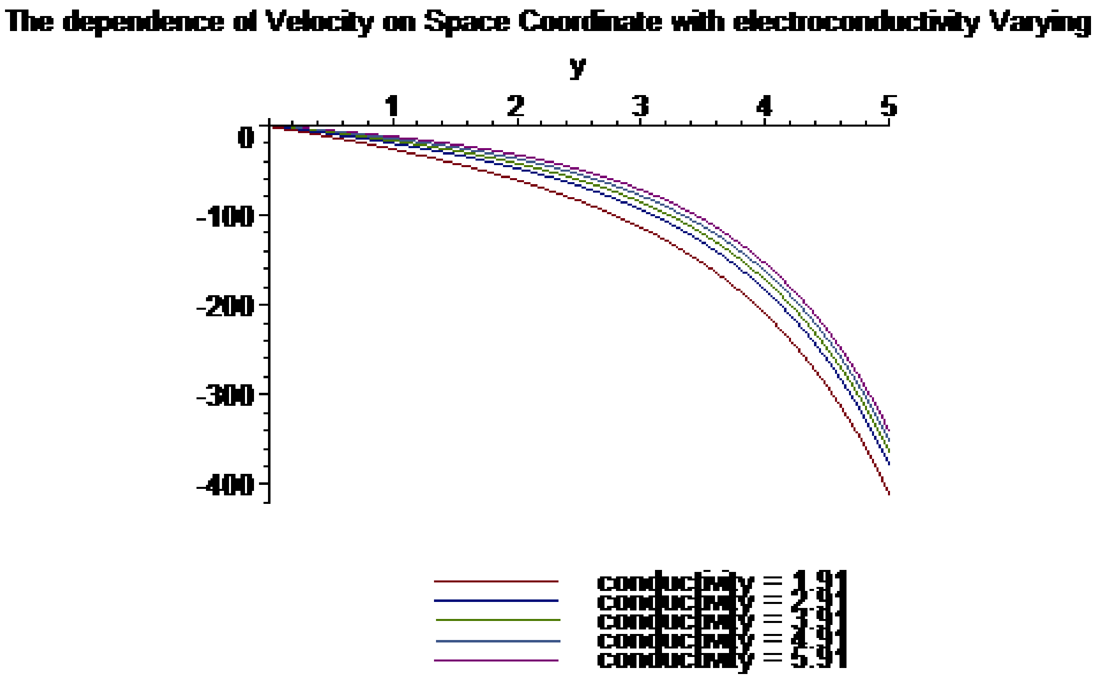

Figure 5. Velocity profile u against boundary layer y for varying electronegativity σ_0

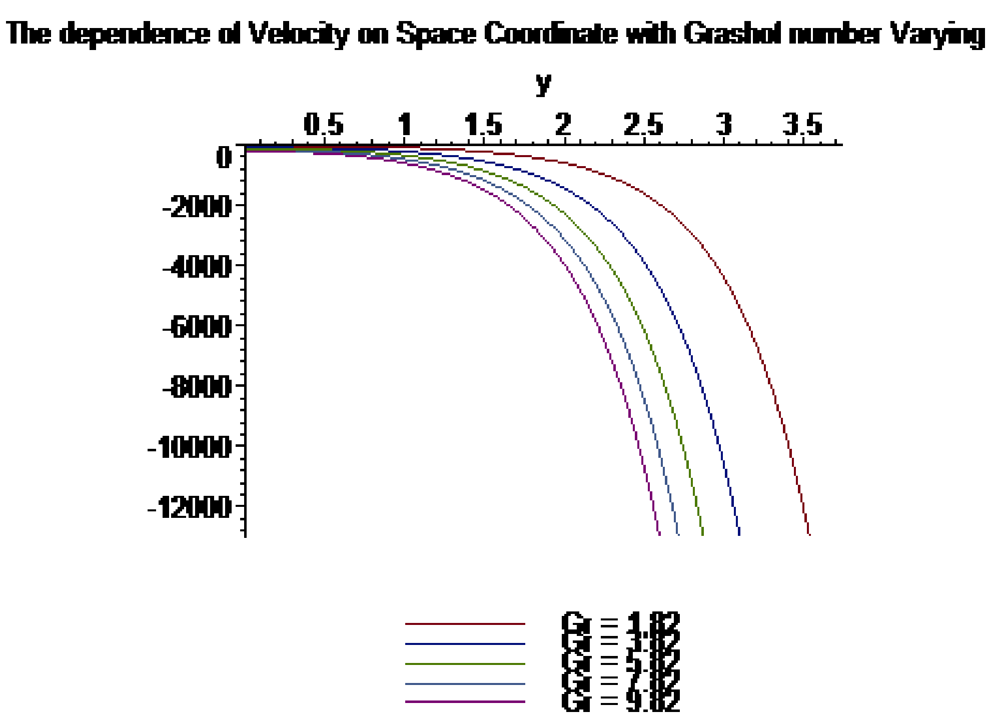

Figure 6. Velocity profile u against boundary layer y for varying Grashof number Gr

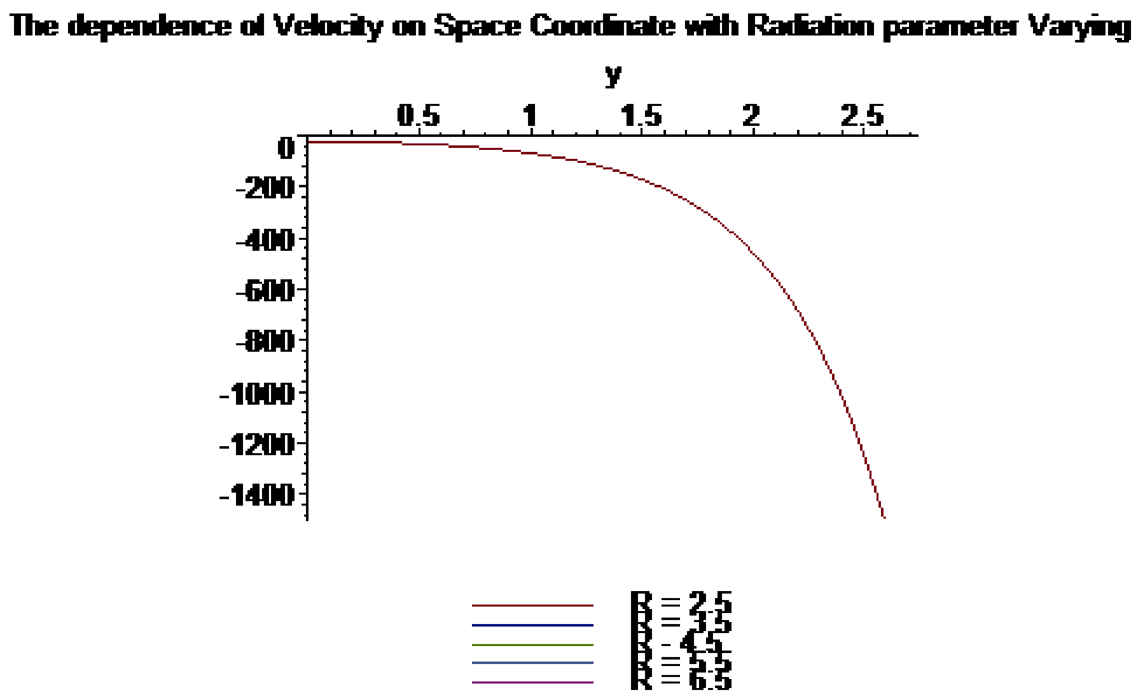

Figure 7. Velocity profile u against boundary layer y for varying Radiation R

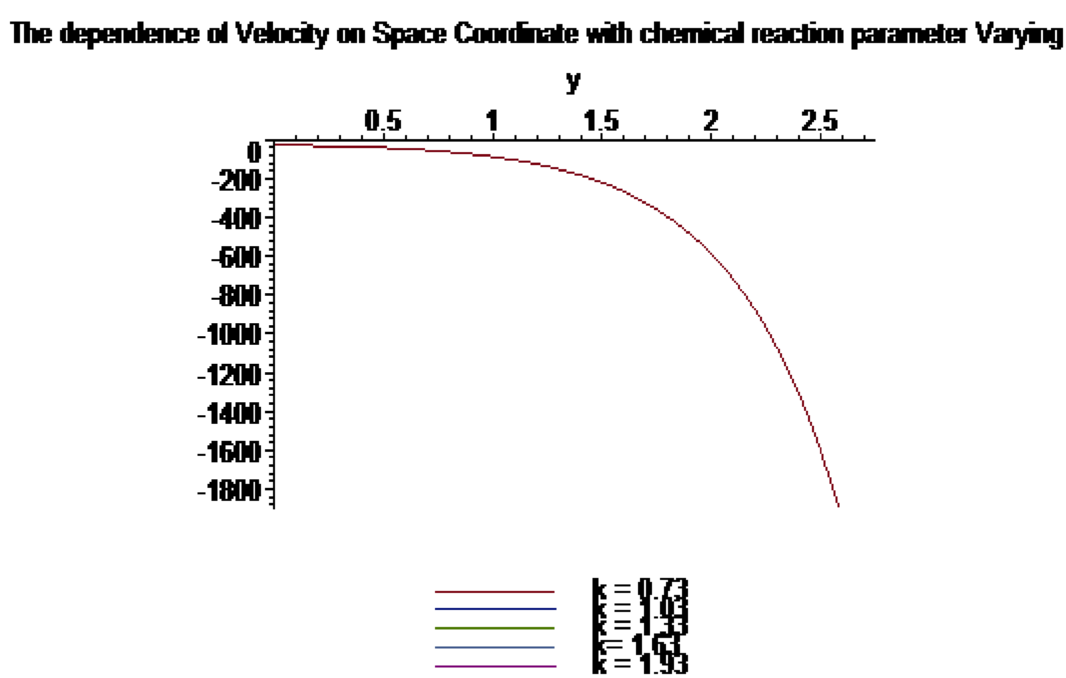

Figure 8. Velocity profile u against boundary layer y for varying Chemical reaction k

This observation is also in line with the studies of [2,4,7,11]. Figure 5, shows increase in electronegativity as a result of sudden movement of the plate and this result in the increase of velocity profile of the fluid and is consistent with the studies of [5] and [10]. Figure 6, shows or describe the free convection currents flow towards the plate as externally heated since the Grashof number depends upon the temperature of the plate hence cooling the plate $Gr > 0$ brigs about decrease in the velocity distribution and this observation is consistent with the findings of [7] and [8]. Figure 7, depicts an increase in radiation which led to an increase in the velocity distribution of the fluid. It also brings about an increase in the shear stresses at the wall of the plate. This assertion is in line with the study of [7]. Figure 8, shows that an increase in the chemical reaction of the fluid result to a marginal increase in the velocity profile of the fluid and this observation is in agreement with the studies of [6] and [7].

5. CONCLUSION

MHD is an essential phenomenon for industrial revolution hence its study in conjunction with other parameters. The interplay of MHD and electronegativity is also very important and play a leading role in the fabrication of several appliances.

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