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On Holography and Statistical Geometrodynamics

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ABSTRACT

The Einstein theory of relativistic gravity encoded in the General Relativity Theory (GRT) is investigated from a holographic statistical geometrophysical viewpoint done so here for the first time. In so doing, the arguments are carried out systematically and the four laws of geometrodynamics are enunciated with a proper reasonable development. To do so, new objects characterizing the quantum geometry christened “geomets” are proposed to exist and it is also proposed that there exist geometrodynamic states that these geomets occupy. The geometrodynamic states are statistical states of different curvature and when occupied determine the geometry of the spacetime domain under scrutiny and thereby tell energy-momentum how to behave and distribute. This is a different take and the theory is developed further by developing the idea that the quantities appearing in the Einstein field equations are in fact physically realistic and measurable quantities called the geometrodynamic state functions. A complete covariant geometrodynamic potential theory is then developed thereafter. Finally a new quantity called the “collapse index” is defined and how the spacetime geometry curves is shown as a first order geometrodynamic phase transition using information bit saturation instead of the concept of temperature. Relationship between purely and only geometry and information is stressed throughout. The statistical formula relating curvature and probability is inverted and interpretation is provided. This is followed by a key application in the form of a correspondence between the Euler-Poincaré formula and the proposed “extended Gibbs formula”. In an appendix the nub of the proof of the Maldacena conjecture is provided.

***Keywords*:** statistical thermodynamics, statistical geometrodynamics, holographic principle, Einstein gravity, bit saturation, chiral geometry

1. INTRODUCTION

The Holographic Principle worked out by 'tHooft and independently by Susskind [1] has played out a vital role in the understanding of gravity. The works of Bousso (see for example for a good survey [2] and also references therein for an exhaustive study of the Holographic Principle) has given a definitive direction to the study of the entropy bounds. Starting from the Bekenstein bound arising from the Geroch process to the work of Susskind through his own process of transforming any thermodynamic system into a blackhole and the application of this transformation to the Bekenstein's Generalized Second Law (GSL) yielding the more rigorous and general spherical entropy bound, one finds a rather subtle and elegant formulation through the light-sheet formalism to the Bousso covariant entropy conjecture (also look up in [2] in the reference section under B for Bousso), now a theorem due to the proof provided by Flanagan et. al. [3]. Recently, Verlinde has made a revelation in gravitational physics [4] by demonstrating the emergence of Newtonian and Einsteinean gravity in steps of approximations from the laws of thermodynamics by the application of the Holographic Principle and bit dynamics. The work of Caticha [5] (check also references in [5]) has taken definitive steps in the direction of construction of a statistical theory of geometrodynamics. The present work takes a completely different route to derive the Einstein gravity and further extends the theory. The geometrodynamic theory is considered from statistical postulates applied to hypothetical fundamental constituents of curved spacetime. The theory of geometrodynamics thus derived is considered independent of thermodynamics and yet a similar theory in its own right with a set of quantities analogous to entropy, thermodynamic probability, temperature, etc. There is thus no such entropy bound here. Everything is in terms of curvature and statistical geometrodynamics. The paper is a bold attempt to pave way in the positive direction of a fundamental understanding of the origins of space and time [6].

2. FULL TEXT: GEOMETRY AND PROBABILITY

The bit dynamics on the holographic screen bounding the bulk spacetime manifold determines the geometry in the bulk. Basically, the binding holographic principle just stated and the endeavor to relate only spacetime geometry and information compelled the author to take up the writing of this paper; being provoked by certain non geometric assumptions of Erick Verlinde in his paper to derive Einstein's General Relativity and the concommitant four laws that emerge in the following follows naturally from a clear reasoning from gravitational physics and information theory. The rest of the development then follows. The theory of quantum gravity should have a statistical basis because quantum mechanics has a statistical basis. Thermodynamics is the statistical mechanical approach to matter. Temperature is a thermodynamic quantity. Where does temperature equivalent to acceleration called the Unruh temperature arise from when the question is about quantum geometry? The principle of equivalence asserts the equivalence of acceleration and gravitation. So, gravitation should record an additional temperature owing to the force of gravity.

This is not observed even in ordinary laboratories on the surface of the earth. And the quantum theory of gravity should as a first step start with a statistical mechanical approach to gravity, yielding a geometry and probability realtionship. The endeavour of the Holographic

Principle to catch ahold of relationship between geometry and information theory, should as a rule do away with the energy-momentum quantum fields and bring in directly information bit dynamics. The quantum field theory is a quantum theory of fields motivated by the success of the quantum theory of the electromagnetic field. The extensions thereafter ensued with the success of the gauge principle and symmetry breaking. But the gravitational field is not an ordinary field theory. Here the field is the curvature of the physically realistic spacetime and not the curvature of the vector bundle associated with the particle quantum of the gauge field. To use the methods of the conventional quantum field theory of energy-momentum to quantize gravity will create anomalies and conceptual difficulties. The following effort is to circumvent these problems and show a novel appraoch to a fundamental understanding of the spacetime geometry and the realtionship of this to the information theory.

The quantum fields can thereafter be related to this theory using the relational machinery that has been developed in the formulation of the laws of statistical geometrodynamics. The principle of equivalence is the only guiding principle to the formulation of general covariance and the concomittant general relativity theory (GRT). So, if the theory of general relativity is indeed entropic as shown by Verlinde, then it is naturally statistical. The main aim here is to show how the Einstein theory can arise as a first statement of the theory of geometrodynamics when statistical methods are applied to a hypothetical ensemble of a conglomeration of quanta of spacetime geometry.

2. 1. THE FOUR LAWS OF GEOMETRODYNAMICS WITH HOLOGRAPHY: A STATISTICAL APPROACH

One always arrives at the Einstein field equations and his law of gravitation through the Principle of equivalence which asserts the equivalence of acceleration and gravitation. Then, there is the Einstein-Hilbert variational principle from which also one gets the Einstein field equations of gravitation which read

$$\boldsymbol{R_{\mu\nu}} - \frac{1}{2}\boldsymbol{g_{\mu\nu}R} = \boldsymbol{T_{\mu\nu}}, \tag{1}$$

where, $\boldsymbol{R_{\mu\nu}}$ is the Ricci curvature tensor, $\boldsymbol{R}$ the corresponding scalar, $\boldsymbol{g_{\mu\nu}}$ is the fundamental or metric tensor and $\boldsymbol{T_{\mu\nu}}$ is the energy momentum tensor. The above equation embodies the fact that spacetime is curved by the presence of matter and mass is made to move by the curved spacetime according to the warp of the spacetime.

This involves tensor analytic manipulation of the Christoffel symbols of the second kind thereby defining the Ricci tensor. Yet another approach is that of the Bianchi identity of the second type which exhibit the principle of geometrodynamics that the boundary of a boundary of a boundary is zero. All these approaches involve deterministic and generically predictive propositions and interpretations. On the other hand, energy-momentum tensor represents matter and energy and these obey quantum statistical laws. There are quantum transitions involved in the matter represented by $\boldsymbol{T_{\mu\nu}}$.

This quantum behavior should correspondingly be accounted for by the spacetime geometry as well. If matter fluctuates statistically then so should geometry. If matter obeys statistical laws then so should geometry at the quantum scales. The laws of physical statistics obeyed by $\boldsymbol{T_{\mu\nu}}$ lead to statistical thermodynamics.

The laws of physical statistics obeyed by the geometric part of the Einstein law viz., the l.h.s. of eq (1) should lead to statistical geometrodynamics. Still, it is the aim of this paper to not argue this way by starting with the Einstein law and the eq (1). The aim is to rather start with a subtle and simple set of statistical postulates and ansatz and arrive at various results.

We begin by proposing the existence of geomets – hypothetical quantum or fundamental objects of spacetime geometry which occupy different available geometrodynamic states in the statistical manner of speaking. The various geometrodynamic distributions are then arrived at by the gas of geomets.

The resulting spacetime geometry is a direct consequence of the ensemble of the gas of geomets occupying the geometrodynamic states of different curvature probabilities. This is our first ansatz. So, now there are two hypothetical objects – (i) geomets – the fundamental constituents of curved spacetime and (ii) the geometrodynamic states. Since entropy for gravity as a thermodynamic system is non-concave, there are many possible geometrodynamic states and end-states. As such, the geometrodynamic probability and the geometrodynamic distribution function for a given ensemble of a gas of geomets determining the curvature in the bulk of spacetime become $\boldsymbol{N^2 = 4^2 = 16}$ functions and are tensors of rank two. We denote them respectively by $\boldsymbol{\Gamma_{\mu\nu}}$ and $\boldsymbol{f_{\mu\nu}}$. Even though the number, $\boldsymbol{n_s}$, of the bits available for the description of the individual species of geomets is the same (is equivalent to the number of matter or radiation particles) the non-concave nature of entropy for gravity makes these tensors. Now from our simple hypothesis, we derive: (i) the Einstein law of geometrodynamics and follow it up with two additional laws based on the holographic principle in analogy with the laws of thermodynamics and (ii) we derive a formula connecting the Gauss-Bonnet mean curvature of the holographic surface bounding the bulk of the spacetime and the geometrodynamic probability $\boldsymbol{\Gamma_{\mu\nu}}$ in the bulk.

The quantity $\boldsymbol{\beta}$ appearing in statistics is inversely proportional to the absolute temperature $\boldsymbol{T}$ in statistical thermodynamics. In statistical geometrodynamics, we define the geometrodynamic probability $\boldsymbol{\Gamma_{\mu\nu}}$ by

$$\boldsymbol{ln\Gamma_{\mu\nu} = \sum_s f_{\mu\nu}(n_s)} \tag{2}$$

Now, we fix the following ansatz,

$$\boldsymbol{\sum_s n_s = N} \quad \text{and} \quad \boldsymbol{\sum_s n_s \gamma_{\mu\nu(s)} = T_{\mu\nu}} \tag{3}$$

Here, $\boldsymbol{\gamma_{\mu\nu(s)}}$ is the kinetic geometry of the species of the geomets. The kinetic geometry is defined as the geometry possessed by the geomet on account of its motion. This is a simple definition: moving bodies possess kinetic energy and moving elements of curved spacetime geometry – the geomets – possess kinetic geometry. This fixes up an exact yet abstruse analogy between thermodynamics and geometrodynamics and further strengthens ansatz (3) above. Pure statistical geometrodynamics should be geometrical in character. Any energy should be translated into geometry and vice-versa. In fact, the first condition of the ansatz eq (3) is simply the number conservation but the second condition of the ansatz (3) is the Principle of Equivalence of Gravitation and Inertia in disguise if one thinks carefully.

So, we fix up tensor multipliers as,

$$\boldsymbol{\alpha_{\mu\nu}} + \boldsymbol{\beta\gamma_{\mu\nu(s)}} = \frac{\partial \boldsymbol{f_{\mu\nu}}}{\partial \boldsymbol{n_s}}. \tag{4}$$

Then,

$$\boldsymbol{\delta ln\Gamma_{\mu\nu}} = \sum_s \frac{\partial \boldsymbol{f_{\mu\nu}}}{\partial \boldsymbol{n_s}} \boldsymbol{\delta n_s} = \sum_s (\boldsymbol{\alpha_{\mu\nu}} + \boldsymbol{\beta\gamma_{\mu\nu(s)}})\boldsymbol{\delta n_s}$$
$$= \boldsymbol{\alpha_{\mu\nu}} \sum_s \boldsymbol{\delta n_s} + \boldsymbol{\beta} \sum_s \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s} \tag{5}$$

Since $\boldsymbol{n_s}$ is fixed by ansatz (3),

$$\sum_s \boldsymbol{\delta n_s} = 0. \tag{6}$$

and from the second part of (3),

$$\sum_s \boldsymbol{n_s} \delta\boldsymbol{\gamma_{\mu\nu(s)}} + \sum_s \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s} = \delta T_{\mu\nu}. \tag{7}$$

The first term in (7) represents the stretch in the spacetime; that is, the geometric work or in other words, the stressing curvature mathematically equal to "$-\frac{1}{2}Rg_{\mu\nu}$"; that is

$$\sum_s \boldsymbol{n_s} \delta\boldsymbol{\gamma_{\mu\nu(s)}} = \sum_s \boldsymbol{n_s} \frac{\partial \gamma_{\mu\nu(s)}}{\partial R} \delta R = -\frac{1}{2} g_{\mu\nu} \delta R, \tag{8}$$

so that

$$g_{\mu\nu} = -2 \sum_s \boldsymbol{n_s} \frac{\partial \gamma_{\mu\nu(s)}}{\partial R}. \tag{9}$$

Also taking local derivatives w.r.t. time we have by Hamilton's Ricci flow

$$\frac{\partial g_{\mu\nu}}{\partial t} = -2 \sum_s \boldsymbol{n_s} \frac{\partial^2 \gamma_{\mu\nu(s)}}{\partial R \partial t} = -2R_{\mu\nu}(g) \tag{10}$$

So

$$\sum_s n_s \frac{\partial^2 \gamma_{\mu\nu(s)}}{\partial R \partial t} = R_{\mu\nu}(g). \tag{11}$$

Now, the second term – "$\sum_s \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s}$" is the Ricci curvature which is the geometrodynamic "warp" accrued by the bulk of the spacetime, i.e., the curvature inside the holographic screen. Thus,

$$\boldsymbol{\delta R_{\mu\nu}} = \sum_s \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s}. \tag{12}$$

From (7), (8) and (12), we have for the first law of geometrodynamics, the Einstein field equations (1) which we rewrite here as

(1) The first law of geometrodynamics: $\boldsymbol{T_{\mu\nu}} = \boldsymbol{R_{\mu\nu}} - \frac{1}{2} \boldsymbol{g_{\mu\nu} R}$.

Also,

$$\sum_s \boldsymbol{n_s} \frac{\partial^2 \gamma_{\mu\nu(s)}}{\partial R \partial t} = \sum_s \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s}$$

Or

$$\boldsymbol{n_s} \frac{\partial^2 \gamma_{\mu\nu(s)}}{\partial R \partial t} = \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s}. \tag{13}$$

On the other hand,

$$\delta ln\Gamma_{\mu\nu} = \beta \sum_s \boldsymbol{\gamma_{\mu\nu(s)}} \boldsymbol{\delta n_s} = \beta \delta R_{\mu\nu}. \tag{14}$$

Therefore, $\beta \delta R_{\mu\nu}$ is a total differential and β, the intergrating factor of the Ricci curvature. The statistical theory leads naturally to the second law of geometrodynamics which we enunciate as follows:

(2) $\boldsymbol{\delta R_{\mu\nu}}$ has an integrating factor namely,

$$\boldsymbol{\delta K_{\mu\nu}} = (\boldsymbol{1/N_{sat}})\boldsymbol{\delta R_{\mu\nu}}, \tag{15}$$

where, $\boldsymbol{K_{\mu\nu}}$ is the Gauss-Bonnet mean curvature of the horizon or the holographic screen/surface. N_{sat} is the bit saturation. Bit saturation is the ratio of the number of bits required to describe the system, in the bulk of the space time including the bulk space time itself, N_{bulk}, to the number of bits available on the holographic screen, N_{surf}. Thus, we have $\boldsymbol{\delta K_{\mu\nu}} \propto N_{surf}$ and $\boldsymbol{R_{\mu\nu}} \propto N_{bulk}$, so that the above law follows by the definition of the bit saturation.

Hence, for quantum gravitational nature of the theory and for dimensional consistency, we insert the Planck length, λ_{Pl}, as

$$\beta = \frac{1}{\lambda_{Pl} N_{sat}}. \tag{16}$$

So,

$$\delta ln\Gamma_{\mu\nu} = \frac{1}{\lambda_{Pl} N_{sat}} \delta R_{\mu\nu}, \tag{17}$$

or

$$K_{\mu\nu} = \lambda_{Pl} ln\Gamma_{\mu\nu}. \tag{18}$$

According to Verlinde [4], $N_{sat} = -\frac{\phi}{2c^2}$ where ϕ represents the Newtonian gravitational potential. This N_{sat} is a positive number that vanishes at large distances. So, this coincides with the fact that the temperature also decreases with the expanding universe. Therefore, we extend the analogy with the third law of thermodynamics and propose that:

(3) The Gauss-Bonnet mean curvature $K_{\mu\nu}$ of the evolving holographic surface tends to zero for pure gravity as the bit saturation N_{sat} tends to zero and becomes zero at zero bit saturation for the pure gravity situation.

Thus, we now have three laws of statistical geometrodynamics. The first law is the Einstein law of gravitation as given by the Einstein field equations. The second law connects the Ricci curvature (tensor) in the Einstein theory to the holographic screen/surface encompassing the Einstein bulk spacetime. And the third law is (more or less) purely about the holographic surface enclosing the Einstein bulk. Another statement for the third law of thermodynamics is that,

"in every irreversible thermodynamic process, the total entropy of the universe always increases". Similarly,

(3') In every irreversible geometrodynamic process, the Gauss-Bonnet mean curvature of the holographic screen bounding the bulk spacetime always increases.

Now, for empty spacetime, the number of bits available in the bulk will be constant and that on the holographic screen will also stay the same. So, we have the zeroeth law of geometrodynamics, viz.,

(0) The bit saturation for a system of empty spacetime bulk bounded by a holographic screen, is a constant.

Just as temperature is a concept that holds macroscopically and breaks down at the individual molecular/atomic level, the bit saturation is a concept that plays out a similar role. Now, temperature is defined as the average kinetic energy of all the particles in a thermodynamic system. At the particle level, there is the individual kinetic energy of the moving particles. For the whole bulk of spacetime and its enclosing Holographic screen, the bit saturation reduces to individual bits and thereby plays out an equivalent role.

The definite physical boundaries of the object (matter) are blurred in the fine graining limit and vanish completely in the completely fine grained structure. Similarly, the spacetime manifold defined by the l.h.s. of (1) will also disappear at the quantum scales where the structure is sufficiently finely grained. Actually the ansatz (3) somehow seems to show a fundamental equivalence between quantum geometry and quantum matter and radiation by establishing an exact relationship between quanta of geometry and quanta of matter. Moreover, the third law leads to an interesting consequence – the AdS/CFT correspondence or the Maldacena conjecture.

In fact, it allows all the similar conjectures such as the Strominger dS/CFT correspondence and the Rehren duality or the Algebraic Holography to follow from it. We shall see how in the Appendix A at the end of the paper. On the other front an important thing to note: from equations (15), (17) and (18), we should speak of probable curvature. That the curvature probably increases in the course of an irreversible geometrodynamic process. That spacetime geometry is probably curved. T

he characteristic Planck length appearing in eqs (17) and (18) imply this to be so for significantly fine graining limit.

2. 2. GEOMETRODYNAMIC POTENTIALS AND THE MAXWELL GEOMETRODYNAMIC RELATIONS

Just as in the case of thermodynamics we identify geometrodynamic state functions. By geometrodynamic state, we mean a configuration of coarse grained spacetime geometry defined by a certain curvature. Each geometrodynamic state has a well-defined curvature. The geomets occupy the various geometrodynamic states and yield a certain curvature distribution delivering a spacetime geometry that tells the matter or the energy-momentum distribution how to distribute and how to behave.By geometrodynamic state functions, we identify with the quantities appearing in the statement eq (1) of the first law of geometrodynamics as well as the new quantities that we shall define below.Now, by the first law we have

$$\boldsymbol{T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}. \tag{1}$$

By the second law, we have

$$\boldsymbol{\delta K_{\mu\nu} = (1/N_{sat}) \delta R_{\mu\nu}} \tag{15}$$

or therefore

$$\boldsymbol{N_{sat} \delta K_{\mu\nu} = \delta R_{\mu\nu}} \tag{15-a}$$

Thus,

$$\boldsymbol{\delta T_{\mu\nu} = N_{sat} \delta K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \delta R} \tag{19}$$

Given the background as thus, we now proceed to further theoretical investigations of the paper. Then, from eq (19),

$$\left(\frac{\partial \boldsymbol{T_{\mu\nu}}}{\partial \boldsymbol{K_{\mu\nu}}}\right)_R = \boldsymbol{N_{sat}}, \qquad \left(\frac{\partial \boldsymbol{T_{\mu\nu}}}{\partial R}\right)_{\boldsymbol{K_{\mu\nu}}} = -\frac{1}{2} \boldsymbol{g_{\mu\nu}}. \tag{20}$$

Since, $\boldsymbol{\delta T_{\mu\nu}}$ is a perfect differential,

$$\left(\frac{\partial}{\partial R}\left(\frac{\partial \boldsymbol{T_{\mu\nu}}}{\partial \boldsymbol{K_{\mu\nu}}}\right)_R\right)_{\boldsymbol{K_{\mu\nu}}} = \left(\frac{\partial}{\partial \boldsymbol{K_{\mu\nu}}}\left(\frac{\partial \boldsymbol{T_{\mu\nu}}}{\partial R}\right)_{\boldsymbol{K_{\mu\nu}}}\right)_R \tag{21}$$

$$\left(\frac{\partial \boldsymbol{N_{sat}}}{\partial R}\right)_{\boldsymbol{K_{\mu\nu}}} = -\frac{1}{2}\left(\frac{\partial \boldsymbol{g_{\mu\nu}}}{\partial \boldsymbol{K_{\mu\nu}}}\right)_R. \tag{22}$$

$$\boldsymbol{N_{sat} \delta K_{\mu\nu} = \delta (N_{sat} K_{\mu\nu})}, \tag{23-a}$$

or,

$$\boldsymbol{\delta(T_{\mu\nu} - N_{sat}K_{\mu\nu}) = -\frac{1}{2}g_{\mu\nu}\delta R} \tag{23-b}$$

therefore giving

$$\boldsymbol{\delta A_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}\delta R}. \tag{24}$$

Here, $\boldsymbol{A_{\mu\nu}}$ is the additional available geometry, where

$$\boldsymbol{A_{\mu\nu} = T_{\mu\nu} - N_{sat}K_{\mu\nu}}\ . \tag{25}$$

This additional available geometry makes its presence felt during cases such as *frame dragging* or *ergosphere* of a Kerr or Kerr-Newman blackhole. A *state of geometrodynamic equilibrium* means a state of *constant curvature* or that of *locally Lorentzian spacetime*. For this to occur, $\boldsymbol{T_{\mu\nu}}$ must be inertial matter and $\boldsymbol{A_{\mu\nu} < \frac{1}{4}T_{\mu\nu}}$ must be the observed key condition. This yields stable geometrodynamic equilibrium. Consider

$$\begin{aligned}\boldsymbol{\delta T_{\mu\nu}} &\boldsymbol{= N_{sat}\delta K_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\delta R} \\ &\boldsymbol{= \delta\left(N_{sat}K_{\mu\nu}\right) - K_{\mu\nu}\delta N_{sat} - \frac{1}{2}g_{\mu\nu}\delta R}\end{aligned} \tag{26-a}$$

$$\boldsymbol{\delta A_{\mu\nu} = -K_{\mu\nu}\delta N_{sat} - \frac{1}{2}g_{\mu\nu}\delta R}\ . \tag{26-b}$$

Now, consider $\boldsymbol{N_{sat}}$ and $\boldsymbol{R}$ as independent variables, then $\boldsymbol{A_{\mu\nu} = A_{\mu\nu}(N_{sat}, R)}$. Therefore,

$$\boldsymbol{\delta A_{\mu\nu} = \left(\frac{\partial A_{\mu\nu}}{\partial N_{sat}}\right)_R \delta N_{sat} + \left(\frac{\partial A_{\mu\nu}}{\partial R}\right)_{N_{sat}} \delta R}\ . \tag{26-c}$$

Whence,

$$\boldsymbol{\left(\frac{\partial A_{\mu\nu}}{\partial N_{sat}}\right)_R = -K_{\mu\nu}}\ , \quad \boldsymbol{\left(\frac{\partial A_{\mu\nu}}{\partial R}\right)_{N_{sat}} = -\frac{1}{2}g_{\mu\nu}}\ . \tag{26-d}$$

$\boldsymbol{\delta A_{\mu\nu}}$ being a perfect differential, we have

$$\left(\frac{\partial}{\partial R}\left(\frac{\partial A_{\mu\nu}}{\partial N_{sat}}\right)_R\right)_{N_{sat}} = \left(\frac{\partial}{\partial N_{sat}}\left(\frac{\partial A_{\mu\nu}}{\partial R}\right)_{N_{sat}}\right)_R \tag{26-e}$$

$$\boldsymbol{\left(\frac{\partial K_{\mu\nu}}{\partial R}\right)_{N_{sat}} = -\frac{1}{2}\left(\frac{\partial g_{\mu\nu}}{\partial N_{sat}}\right)_R}\ . \tag{27}$$

Now,

$$\boldsymbol{\frac{1}{2}g_{\mu\nu}\delta R = \delta\left(\frac{1}{2}g_{\mu\nu}\delta R\right) - \frac{1}{2}R\,\delta g_{\mu\nu}} \tag{28-a}$$

$$\delta T_{\mu\nu} = N_{sat}\delta K_{\mu\nu} - \delta\left(\frac{1}{2} g_{\mu\nu} R\right) + \frac{1}{2} R\, \delta g_{\mu\nu} \tag{28-b}$$

So,

$$\delta\left(T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R\right) = N_{sat}\delta K_{\mu\nu} + \frac{1}{2} R\, \delta g_{\mu\nu} \tag{28-c}$$

and since $T = -R$, we have

$$\delta H_{\mu\nu} = N_{sat}\delta K_{\mu\nu} + \frac{1}{2} R\, \delta g_{\mu\nu}\ , \tag{29}$$

where

$$H_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \tag{30}$$

is the geometrodynamic equivalent of enthalpy in thermodynamics and thermal field theory. Let $H_{\mu\nu} = H_{\mu\nu}(K_{\mu\nu}\,, g_{\mu\nu})$. Then,

$$\delta H_{\mu\nu} = \left(\frac{\partial H_{\mu\nu}}{\partial K_{\mu\nu}}\right)_{g_{\mu\nu}} \delta K_{\mu\nu} + \left(\frac{\partial H_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{K_{\mu\nu}} \delta g_{\mu\nu}\ . \tag{31-a}$$

Comparing eqs (30) and (31)

$$\left(\frac{\partial H_{\mu\nu}}{\partial K_{\mu\nu}}\right)_{g_{\mu\nu}} = N_{sat}\ , \quad \left(\frac{\partial H_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{K_{\mu\nu}} = \frac{1}{2} R\ . \tag{31-b}$$

Now, since $\delta H_{\mu\nu}$ is a perfect differential,

$$\left(\frac{\partial}{\partial g_{\mu\nu}}\left(\frac{\partial H_{\mu\nu}}{\partial K_{\mu\nu}}\right)_{g_{\mu\nu}}\right)_{K_{\mu\nu}} = \left(\frac{\partial}{\partial K_{\mu\nu}}\left(\frac{\partial H_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{K_{\mu\nu}}\right)_{g_{\mu\nu}}\ . \tag{31-c}$$

and hence,

$$\left(\frac{\partial N_{sat}}{\partial g_{\mu\nu}}\right)_{K_{\mu\nu}} = \frac{1}{2}\left(\frac{\partial R}{\partial K_{\mu\nu}}\right)_{g_{\mu\nu}}\ . \tag{32}$$

Consider now the $N_{sat}\delta K_{\mu\nu}$ equation

$$\begin{aligned} \delta T_{\mu\nu} &= N_{sat}\delta K_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\delta R \\ &= \delta(N_{sat}K_{\mu\nu}) - K_{\mu\nu}\delta N_{sat} - \delta\left(\frac{1}{2} g_{\mu\nu} R\right) + \frac{1}{2} R\, \delta g_{\mu\nu}\ , \end{aligned} \tag{33-a}$$

or

$$\boldsymbol{\delta E_{\mu\nu} = -K_{\mu\nu}\delta N_{sat} + \frac{1}{2}R\, \delta g_{\mu\nu}} \,. \tag{33-b}$$

where we have introduced

$$\boldsymbol{E_{\mu\nu} = T_{\mu\nu} - N_{sat}K_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R} \,, \tag{34}$$

or using eqs (29) and (30)

$$\boldsymbol{E_{\mu\nu} = H_{\mu\nu} - N_{sat}K_{\mu\nu}} \,. \tag{35}$$

$\boldsymbol{E_{\mu\nu}}$ is what we dub as the Einstein free geometry. It is the geometry that keeps energy-momentum distributions from collapsing or the spacetime geometry itself from undergoing gravitational collapse. It provides with extra geometry when the Einstein free geometry is discontinuous, a collapse can ensue as will be seen in the next section. From eq (25) using the expression for the additional available geometry, $\boldsymbol{E_{\mu\nu}}$ can also be written as

$$\boldsymbol{E_{\mu\nu} = A_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R} \,. \tag{36}$$

Take $\boldsymbol{N_{sat}}$ and $\boldsymbol{g_{\mu\nu}}$ as independent variables, whence $\boldsymbol{E_{\mu\nu} = E_{\mu\nu}(N_{sat}, g_{\mu\nu})}$. Therefore,

$$\boldsymbol{\delta E_{\mu\nu} = \left(\frac{\partial E_{\mu\nu}}{\partial N_{sat}}\right)_{g_{\mu\nu}} \delta N_{sat} + \left(\frac{\partial E_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{N_{sat}} \delta g_{\mu\nu}} \,. \tag{37-a}$$

Comparing, we have

$$\boldsymbol{\left(\frac{\partial E_{\mu\nu}}{\partial N_{sat}}\right)_{g_{\mu\nu}} = -K_{\mu\nu} \,, \quad \left(\frac{\partial E_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{N_{sat}} = \frac{1}{2}R} \,. \tag{37-b}$$

Now, $\boldsymbol{\delta E_{\mu\nu}}$ being a perfect differential,

$$\boldsymbol{\left(\frac{\partial}{\partial g_{\mu\nu}}\left(\frac{\partial E_{\mu\nu}}{\partial N_{sat}}\right)_{g_{\mu\nu}}\right)_{N_{sat}} = \left(\frac{\partial}{\partial N_{sat}}\left(\frac{\partial E_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{N_{sat}}\right)_{g_{\mu\nu}}} \,. \tag{37-c}$$

Following which by eqs (37-b) and (37-c)

$$\boldsymbol{\left(\frac{\partial K_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{N_{sat}} = -\frac{1}{2}\left(\frac{\partial R}{\partial N_{sat}}\right)_{g_{\mu\nu}}} \,. \tag{38}$$

These geometrodynamic relations (22), (27), (32) and (38) are the direct consequence of the fact that the geometrodynamic variables $\boldsymbol{N_{sat}}$, $\boldsymbol{K_{\mu\nu}}$, $\boldsymbol{g_{\mu\nu}}$ and $\boldsymbol{R}$ are not completely independent but are related through the fundamental geometrodynamic relations which we conventionally refer to as the Einstein Field Equations of (relativistic) gravitation or in other words General Relativity (GR), viz.,

$$\boldsymbol{\delta T_{\mu\nu} = \delta R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\delta R} \ . \qquad (39)$$

and we have

$$\boldsymbol{\delta T_{\mu\nu} = N_{sat}\delta K_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\delta R} \ . \qquad (19)$$

and we have

$$\boldsymbol{\left(\frac{\partial R_{\mu\nu}}{\partial R}\right)_{N_{sat}} = N_{sat}\left(\frac{\partial g_{\mu\nu}}{\partial N_{sat}}\right)_{R}}. \qquad (40)$$

If we define the curvature capacities at constant metric and constant Ricci scalar as

$$C_{g_{\mu\nu}} = \boldsymbol{\left(\frac{\partial R_{\mu\nu}}{\partial N_{sat}}\right)}_{g_{\mu\nu}} \text{ and } C_R = \boldsymbol{\left(\frac{\partial R_{\mu\nu}}{\partial N_{sat}}\right)}_{R}. \qquad (41)$$

Define further $\frac{C_{g_{\mu\nu}}}{C_R} = \eta$, the curvature-bit index. For, a perfect gas of geomets, we can assume an "ideal gas" equation,

$$\boldsymbol{\frac{1}{2} g_{\mu\nu} R = \lambda_{Pl} N_{sat} \delta_{\mu\nu}} \qquad (42)$$

What is a perfect gas of geomets? If $\boldsymbol{\frac{1}{2} g_{\mu\nu} R = \mathrm{k} N_{sat}}$ is the ideal gas or rather ideal geometry equation then $\boldsymbol{N_{sat}}$ is just a number and $\boldsymbol{\frac{1}{2} g_{\mu\nu} R}$ has the dimensions of length so that the only plausible constant for $\boldsymbol{k}$ **is** $\boldsymbol{k = \lambda_{Pl}\delta_{\mu\nu}}$ which leads us to the above eq(42). The presence of $\boldsymbol{\delta_{\mu\nu}}$ in the r.h.s. makes the l.h.s. a symmetric tensor quantity. Thus, ideal geometry is that of a symmetric or even isotropic geometric configuration, i.e., isotropic spacetime geometry or a flat Lorentzian spacetime geometry. Now, isotropy implies homogeneity so, an isotropic and homogeneous (Lorentzian) spacetime geometry represents an ideal geometry. This should persist right up to the Planck scale. When various energy momentum related physical processes take place, some deviations from the ideal behavior are naturally expected. How do these translate at the holographic boundary. The deviation from ideal geometry behavior is a kind of magnetization of the geometry. The test for this is the formation of orbits of smaller momenergy object around the stronger sources of mass-energy, as well as the bending of light. The existence of curved spacetime geometry, is like the existence of a magnetic field and the bending of light is like the passing of the electric current due to the electromotive force (e.m.f.) generated by the magnetization. The fundamental principle is that the ground state of $\boldsymbol{R}$ is not zero, viz., $\langle 0|\boldsymbol{R}|0\rangle \neq 0$.

Similarly, we have

$$\boldsymbol{\left(\frac{\partial R_{\mu\nu}}{\partial g_{\mu\nu}}\right)_{N_{sat}} = -N_{sat}\left(\frac{\partial R}{\partial N_{sat}}\right)_{g_{\mu\nu}}} \qquad (43)$$

The definitions for $C_{g_{\mu\nu}}$ and C_R imply,

$$C_{g_{\mu\nu}} = \boldsymbol{N_{sat}} \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_{\boldsymbol{g_{\mu\nu}}} \text{ and } C_R = \boldsymbol{N_{sat}} \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_R \tag{44}$$

Let us consider $\boldsymbol{N_{sat}}$ and $\boldsymbol{g_{\mu\nu}}$ as independent variables, such that $\boldsymbol{K_{\mu\nu}} = \boldsymbol{K_{\mu\nu}}(\boldsymbol{N_{sat}}, \boldsymbol{g_{\mu\nu}})$. Therefore,

$$\boldsymbol{\delta K_{\mu\nu}} = \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_{\boldsymbol{g_{\mu\nu}}} \boldsymbol{\delta N_{sat}} + \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{g_{\mu\nu}}}\right)_{\boldsymbol{N_{sat}}} \boldsymbol{\delta g_{\mu\nu}} \tag{45-a}$$

$$\boldsymbol{\delta R_{\mu\nu}} = \boldsymbol{N_{sat}}\boldsymbol{\delta K_{\mu\nu}} = \boldsymbol{N_{sat}} \left[\left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_{\boldsymbol{g_{\mu\nu}}} \boldsymbol{\delta N_{sat}} + \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{g_{\mu\nu}}}\right)_{\boldsymbol{N_{sat}}} \boldsymbol{\delta g_{\mu\nu}}\right] \tag{45-b}$$

Using the expression for $C_{g_{\mu\nu}}$, we write

$$\boldsymbol{\delta R_{\mu\nu}} = \boldsymbol{N_{sat}}\boldsymbol{\delta K_{\mu\nu}} = C_{g_{\mu\nu}}\boldsymbol{\delta N_{sat}} + \boldsymbol{N_{sat}} \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{g_{\mu\nu}}}\right)_{\boldsymbol{N_{sat}}} \boldsymbol{\delta g_{\mu\nu}} \tag{45-c}$$

Now, if the metric stays constant $\boldsymbol{\delta g_{\mu\nu}} = 0$. But in calculating C_R, $\boldsymbol{N_{sat}}$ and $\boldsymbol{R}$ are used as the independent variables. To express $\boldsymbol{\delta R_{\mu\nu}}$ in the above equation in terms of $\boldsymbol{\delta N_{sat}}$ and $\boldsymbol{\delta R}$, it is only necessary to express $\boldsymbol{\delta g_{\mu\nu}}$ in terms of these differential quantities. So, we begin with $\boldsymbol{g_{\mu\nu}} = \boldsymbol{g_{\mu\nu}}(\boldsymbol{N_{sat}}, \boldsymbol{R})$ and obtain

$$\boldsymbol{\delta R_{\mu\nu}} = \boldsymbol{N_{sat}}\boldsymbol{\delta K_{\mu\nu}} = C_{g_{\mu\nu}}\boldsymbol{\delta N_{sat}} + \boldsymbol{N_{sat}} \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{g_{\mu\nu}}}\right)_{\boldsymbol{N_{sat}}} \left[\left(\frac{\partial \boldsymbol{g_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_{\boldsymbol{R}} \boldsymbol{\delta N_{sat}} + \left(\frac{\partial \boldsymbol{g_{\mu\nu}}}{\partial \boldsymbol{R}}\right)_{\boldsymbol{N_{sat}}} \boldsymbol{\delta R}\right] \tag{46}$$

The Ricci curvature $\boldsymbol{\delta R_{\mu\nu}}$ or the geometrodynamic heat (curvature) absorbed under conditions where $\boldsymbol{R}$ is constant is then immediately obtained by putting $\boldsymbol{\delta R} = \boldsymbol{0}$. Dividing this "geometric heat" by $\boldsymbol{\delta N_{sat}}$ gives C_R. Thus,

$$C_R = \boldsymbol{N_{sat}} \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_R = C_{g_{\mu\nu}} + \boldsymbol{N_{sat}} \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial \boldsymbol{g_{\mu\nu}}}\right)_{\boldsymbol{N_{sat}}} \left(\frac{\partial \boldsymbol{g_{\mu\nu}}}{\partial \boldsymbol{N_{sat}}}\right)_{\boldsymbol{R}} \tag{47}$$

This is the relation between C_R and $C_{g_{\mu\nu}}$ but it involves quantities like $\left({\partial \boldsymbol{K_{\mu\nu}}}/{\partial \boldsymbol{g_{\mu\nu}}}\right)$ which are not easily computed. We can now use one of the geometrodynamic potential relations to express such quantities.

2. 3. SPACETIME GEOMETRY AS A RICCI PHASE TRANSITION AND CHIRAL GEOMETRY

Consider a ensemble or gas of geomets. Let $\boldsymbol{R_{\mu\nu}}$ be the Ricci curvature resulting from the distribution of the fluid/ensemble of the geomets among the various statistical geometrodynamic states. The bit saturation of the ensemble is defined by

$$B = \boldsymbol{N_{sat}} = \left(\frac{\partial \boldsymbol{K_{\mu\nu}}}{\partial\, \boldsymbol{R_{\mu\nu}}}\right)^{-1}, \tag{48}$$

where $\boldsymbol{K_{\mu\nu}}$ plays the role like the entropy.

Consider the variation of the bit saturation of the fluid of geomets. Let $\boldsymbol{g_{\mu\nu}}$ be the metric of the spacetime and $\boldsymbol{R}$ be the Ricci curvature scalar. If the bit saturation, $\boldsymbol{B}$, of the fluid of geomets is decreased, the liquid condenses to form a rigid causal solid – remember that the spacetime is quite rigid in physically realistic sense and in classical GRT, huge energy-momentum distributions are required to curve it. The curving of the spacetime geometry is the question that we wish to address. How much does a spacetime geometry curve? This is determined by a new parameter that we call the *collapse index*. In our case here the collapse index is the Ricci curvature scalar $\boldsymbol{R}$. The collapse index is the parameter that determines the most probable spacetime geometry or in other words it is the parameter that determines the most probable energy-momentum distribution. The partition function tensor of this gas is

$$\boldsymbol{z_{\mu\nu}} = Tr\; e^{-\beta(\boldsymbol{R}_{\mu\nu(s)} - C_{\mu\nu}N_{(s)})}, \tag{49}$$

where $C_{\mu\nu}$ is the chemical potential variable of the geomet ensemble. The pressure is replaced by the metric. the metric is defined in the coarse graining limit when the geomets exert enough geometric pressure, $\Pi_{\mu\nu}$. The geometrodynamic distribution determining the spacetime geometry is a chiral condensate and is determined as a response of the partition function (49) to a change in the metric as

$$\boldsymbol{R} = -\frac{\partial}{\partial \boldsymbol{g^{\mu\nu}}} ln \boldsymbol{z_{\mu\nu}}. \tag{50}$$

A discontinuity in the Gauss-Bonnett mean curvature

$$\boldsymbol{K_{\mu\nu}} = \left(B\frac{\partial \boldsymbol{\Pi_{\mu\nu}}}{\partial B} - \boldsymbol{\Pi_{\mu\nu}}\right) \tag{51}$$

can be expressed by the gap in the entropy σ,

$$disc\; \sigma = B_c\, Tr\left\{disc\; \frac{\boldsymbol{\partial \Pi_{\mu\nu}}}{\boldsymbol{\partial B}}\right\} \tag{52}$$

From (50) with $\mathbf{ln}\, \boldsymbol{z_{\mu\nu}} = (\boldsymbol{\Pi_{\mu\nu}} - \boldsymbol{K_{\mu\nu(0)}})$ where $\boldsymbol{K_{\mu\nu(0)}}$ is the ground state of the geometry os the holographic boundary spacetime. To this corresponds $\langle 0|\boldsymbol{R}|0\rangle$; it follows that

$$\boldsymbol{R} = \frac{\partial \boldsymbol{K_{\mu\nu(0)}}}{\partial \boldsymbol{g_{\mu\nu}}} - \frac{\partial \boldsymbol{\Pi_{\mu\nu}}}{\partial g_{\mu\nu}} = \langle 0|\boldsymbol{R}|0\rangle - \frac{\partial \Pi_{\mu\nu}}{\partial g_{\mu\nu}}. \tag{53}$$

If we expand the pressure in the vicinity of B_c according to

$$\Pi_{\mu\nu} = \Pi_{\mu\nu(c)} + (B - B_c)\frac{\partial \Pi_{\mu\nu}}{\partial B}\bigg|_{B_c} + \;.... \tag{54}$$

$\Pi_{\mu\nu}$ depends on $\boldsymbol{R_{\mu\nu}}$ via $\Pi_{\mu\nu(c)}$ and B_c

$$\frac{\partial \Pi_{\mu\nu}}{\partial g_{\mu\nu}} = \frac{\partial \Pi_{\mu\nu c}}{\partial g_{\mu\nu}} - \frac{\partial B_c}{\partial g_{\mu\nu}} \frac{\partial \Pi_{\mu\nu}}{\partial B}\bigg|_{B=B_c} \tag{55}$$

Inserting (55) into (53) and applying *"disc"* to both the sides, the result is

$$disc\ \boldsymbol{R} = \frac{\partial \boldsymbol{B}_c}{\partial g_{\mu\nu}} disc\ \frac{\partial \Pi_{\mu\nu}}{\partial B}\bigg|_{B=B_c}. \tag{56}$$

While $\Pi_{\mu\nu}$ is discontinuous at B_c (as in the case of a first order phase transition), $\frac{\partial \Pi_{\mu\nu}}{\partial B}$ may jump. From (52) and (56), we finally obtain

$$\frac{\partial B_c}{\partial g_{\mu\nu}} = B_c \frac{disc\ \boldsymbol{R}.}{disc\ \boldsymbol{K}_{\mu\nu}}. \tag{57}$$

Now, $disc\ \boldsymbol{K}_{\mu\nu}$ implies a Gauss-Bonnett mean curvature gap. This is the G-B mean curvature condition of statistical geometrodynamics analogous to the entropy comdition in fluid mechanics. Analogously, therefore, with the G-B mean curvature gap will be associated a phenomenon analogous to the shock waves in fluid dynamics. On the cosmological scales and the large-scale structure of spacetime, it may be the inflationary expansion of the spacetime in the bulk associated with the G-B mean curvature condition. (It is the metric that extends in all directions and hence acts like pressure.)

On the quantum scales it could be formation of bubbles in vacuum due to the quantum shimmering of the virtual quantum particle-antiparticle pairs. The phase transition also defines some kind of chirality due to which the information develops in such a way that the entropy always increases and the universe always expands. The chirality is the very basis of bit dynamics and by the formula given by eq (18) viz., $K_{\mu\nu} = \lambda_{Pl} ln\Gamma_{\mu\nu}$, it is also the very basis of holography and by the relation expressing the second law of geometrodynamics, the very basis of geometrodynamics as well.

The physical prediction of the Ricci phase transition eq (57) is this: a shuffle in the bit does nothing but increment or decrement in the bit density causes shuffles in the geometrodynamic states through the shuffles in the states available for the G-B mean curvature and consequently, in the metric $\boldsymbol{g}_{\mu\nu}$. This makes $\boldsymbol{g}_{\mu\nu}$ and $\boldsymbol{K}_{\mu\nu}$ tensorial in character. Also the Einstein free energy will have a first order discontinuity. Only then can the geometry occur.

However, the avoidance of spacetime singularities as directly suggested by our approach here tells that there should be bound on the latent geometrodynamic heat.

2. 4. A LITTLE BIT ABOUT THE KINETIC GEOMETRY OF THE BIT

Now, the entropy of spin systems and gravity (which has been shown to be a spin system relatively recently by Sen [7] and further development and reformulation of Einstein's GR had been carried out by Ashtekar [8]) is non-concave. Thus, there is no unique thermodynamic state for a given maximized entropy. This is also the case for geometrodynamic state. So, we have a geometrodynamic probability (in a) matrix. Simply, invert the relation (18), as

$$\Gamma_{\mu\nu} = exp\left({K_{\mu\nu}}/{\lambda_{Pl}} \right). \tag{58}$$

It is an exponential of the holographic boundary curvature. The more the curvature, the geometrodynamic probability decreases actually; the Planck length in the exponential is a very small quantity; the detailed arguments and discussion will be the topic of *"irreversible geometrophysics and fluctuation theory"* and will be considered in the second installment of this title.

Any bit fluctuations cause fluctuations in curvature. The elements of the matrix $\Gamma_{\mu\nu}$ correspond to the various apriori probabilities and this leads to different possible geometrodynamic distributions of the geomets for same maximized entropy so $K_{\mu\nu}$ is a path integral in the quantum limit and the geomets can actually undergo a Brownian motion. As these do so, so do the bits on the holographic screen. In short, the geometrodynamic probability changes exponentially with change in the holographic screen curvature.

2. 5. AN APPLICATION OF THE LAWS OF GEOMETRODYNAMICS: THE MODIFIED GIBBS PHASE RULE

The force of gravity by itself occupying the bulk of the spacetime, in the form of its curved geometry, endows globally a simple topology of simply connected spacetime and hence the holographic surface for a pure mass/ energy-momentum based configuration such as a Schwarzschild one is a spherically symmetric one – a hypersphere.

The presence of other coupling constants such as charge, spin, color, etc., lead to more involved and complex topologies of a multiply-connected nature as is observed from the embedding diagrams of the other types of blackholes. This argument is in fact an extension of the embedding diagrams of the complex topological geometries of the Reissner-Nørdstrom, Kerr and the Kerr-Newman blackholes as well as the recent colored blackholes. As such, the Euler-Poincaré theorem for these surfaces is

$$v - e + f = 2(1 - g) = \chi, \tag{59}$$

where, v is the number of vertices of the triangulation of the surface, e is the number of its edges and f is the number of faces formed by the vertices and the incident edges of the triangulation. Eq(59) holds for orientable surfaces. We assume that these are closed orientable holographic surfaces. In general the assumption of a holographic surface will always be that of a closed orientable one.

The corresponding thermodynamics due to gravity acting in conjunction with other forces arising from the non-gravitational properties/ coupling constants should be naturally irreversible and the Gibbs phase rule is a manifestation of this Euler-Poincaré theorem for simple reversible thermodynamic phenomena where the genus of the concerned manifold whose holographic boundary corresponds to the physical boundary of the thermodynamic system is $g = 0$.

This simply yields the Euler rule for convex polyhedral which are the triangulations of any closed simply connected surface. For the irreversible case, the Gibbs phase rule should be

$$F - C + P = 2(1 - k), \tag{60}$$

where, k is the "irreversibility index". F is the number of degrees of freedom of the thermodynamic system, C is the number of components or the dimension of the manifold of coexisting phases and, P, the number of phases.

More complicated, the topology and geometry of the holographic surface in question, more complex the nature of the combined forces at work within the surface and hence more the degree of irreversibility of the thermodynamic process involved. Hence, more the value of the irreversibility index. All this, if more the genus of the holographic surface.

Thus, for the Euler-Poincaré characteristic, $\chi(S)$ for a holographic surface, there corresponds a Gibbs characteristic $\gamma(\Pi)$, where, Π is the phase space for the thermodynamic system under investigation for holographic consideration. This Gibbs characteristic corresponds to the thermodynamics of forces and the matter/ energy-momentum distributions enclosed by the holographic surface S.

To know the details of the degrees of freedom of gravity combined with matter to the degrees of freedom of matter alone, one must take into account the Chern classes. Similar to these, we correspondingly, develop a thermodynamic characteristic classes – a set of "thermodynamic foliations" or "phase foliations".

This will developed in detail in a separate paper. Foe the P thermodynamic phases, the dimension of the manifold of coexisting phases is C, and so in complex dimension C, the highest thermodynamic class is the Cth class. These are the Δ-classes or the phase foliations of as thermodynamic. The discussion of this topic is not meant for the current paper so all this is more or less hand waving. But this is indeed an application of holographic statistical geometrodynamics.

2. 6. APPENDIX

We had surmised earlier that the third law of geometrodynamics can be used to prove the Maldacena conjecture. Let us see how this can be done; but what is to follow is just the nub of the proof of the conjecture and is not to be mistaken as a mathematician's proof of the AdS/CFT correspondence.

Now,

$$\frac{\delta \boldsymbol{K}_{\mu\nu}}{\delta \boldsymbol{R}_{\mu\nu}} = \frac{\boldsymbol{N}_{surf}}{\boldsymbol{N}_{bulk}}, \tag{A-1}$$

but

$$\frac{\boldsymbol{N}_{bulk}}{\boldsymbol{N}_{surf}} = \boldsymbol{N}_{sat} = \boldsymbol{B}, \tag{A-2}$$

Therefore, we have the second law of geometrodynamics as

$$\boldsymbol{\delta K}_{\mu\nu} = \frac{1}{\boldsymbol{B}} \boldsymbol{\delta R}_{\mu\nu}. \tag{A-3}$$

As $\boldsymbol{B} \to \boldsymbol{0}$, the third law of geometrodynamics, $\boldsymbol{K}_{\mu\nu} \to \boldsymbol{0}$ as well. But while this happens, $\boldsymbol{R}_{\mu\nu}$ need not necessarily tend to zero. This is the special case where the closed loop string excitations in the anti-DeSitter space – $\boldsymbol{AdS_5}$ give rise to the gravitational interactions of string (quantum) on the boundary spacetime are naturally non-gravitational. Now, the only possibility for the Ricci being non-vanishing and the $\boldsymbol{K_{\mu\nu}}$ vanishing is when $\boldsymbol{N_{surf}} \to \infty$.

Take this $\boldsymbol{N_{surf}}$ as the number of degrees of freedom of the superconformal gauge field theory and we have an exact correspondence required by the Maldacena conjecture. That is in the large N limit of superconformal field theories defined on the $d = 3 + 1$ boundary spacetime, for the gravitational interactions in the $\boldsymbol{AdS_5}$ bulk of the space, there exist a corresponding such large N superconformal gauge field theories on the Holographic boundary of the $\boldsymbol{AdS_5}$ anti-DeSitter space.

3. CONCLUSIONS

The inclusion of the holographic principle in a systematic manner suggests an extension of the Einstein gravity involving only geometry and information. The understanding of gravitation achieves a deeper meaning in this context and a systematic development follows which takes a different take on the phenomenon of gravitation as an entropic force as demonstrated by Verlinde in [4]. The approach taken here is the usual tensorial but the ideas can be easily applied to say, Ashtekar variables and the enveloping Loop Quantum Gravity and its many modern avatars.

The key insight here is that the inclusion of the holographic principle and its application makes general relativity statistical when endeavors are made to connect geometric gravity to information.as suggested by eq (18). Thus one can speak of the curvature of spacetime as the spacetime will probably curve in this fashion and that the spacetime curvature will probably increase in the course of an irreversible geometrodynamic process such as gravitational collapse.

This is due to the probablistic nature of quantum mechanics that creeps in through the assumption of the quanta of geometry. It would still persevere even if the assumption were that of strings in the non-hadronic dual resonance models or of the spin network states in Loop Quantum Gravity or even simplexes of causal dynamical triangulations. An effectiveness of this approach is the possibility of the sudden validation of the Maldacena conjecture and a new way of looking at different spacetime geometries as due to the occupation index of the gas of the quanta of geometry due to their distribution amongst the various geometrodynamic states which themselves correspond to different curvatures. Elsewhere the quantization of these states and the several ramifications and consequences as well as applications will be discussed in another paper by the author.

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