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Edge Reduced Skolem Difference Mean Number of Some Graphs

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ABSTRACT

A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices x ϵ V with distinct elements $f(x)$ from $\{1,2,3,\ldots,p+q\}$ in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are from $\{1,2,3,\ldots,q\}$. A graph that admits skolem difference mean labeling is called a skolem difference mean graph. In this paper, the author studied the edge reduced skolem difference mean number of some graphs.

***Keywords*:** Skolem difference mean graph, skolem difference mean labeling, edge reduced skolem difference mean number, some graphs

AMS subject classification: 05C78

1. INTRODUCTION

Graphs considered in this paper are finite, undirected and simple. Terms and notations not defined here are used in the sense of Harary [3]. Let $G = (V, E)$ be a graph with p vertices and q edges. $|V(G)| = p$ is called the order of G and $|E(G)| = q$ is called the size of G. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges/ both), then the labeling is called vertex (edge/total) labeling. Rosa introduced $\alpha -$ valuations in [10] which in later called graceful graphs. There are several types of graph labeling and a detailed survey is found in [4]. The concept of skolem difference mean labeling was introduced in [7] and further results were proved in [5,6,9,12,13]. Acharys et.al proved in [2] that any connected graph can be embedded as an induced sub graph of a connected graceful graph. This inspired the author to introduce a new parameter called edge reuced skolem difference mean number for a non skolem difference mean graph. In this paper, the author studied the edge reduced skolem difference mean number of some graphs.

Definition 1.1: Let G_1 and G_2 be two graphs with disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. The join $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all edges joining V_1 with V_2.

Definition 1.2: Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their Cartesian product $G_1 \times G_2$ is defined to be the graph whose vertex set is $V_1 \times V_2$ and edge set is $\{(u_1, v_1), (u_2, v_2)/$ either $u_1 = u_2$ and $v_1 v_2 \in E_2$ or $v_1 = v_2$ and $u_1 u_2 \in E_1\}$.

Definition 1.3[1]: A graph G is said to be embedded in a graph G’, written as G $\prec$ G’, if there exists an induced subgraph of G’ which is isomorphic to G.

Definition 1.4[15]: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing all edges incident to v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.5[11]: For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph $S(G)$ thus obtained is called the splitting graph of G.

Definition1.6[14]: The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition 1.7: A ladder L_n of n steps is the graph $P_n \times P_2$.

Definition 1.8[7]: A graph $G=(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \epsilon V$ with distinct elements $f(x)$ from the set $\{1,2,3,\dots,p+q\}$ in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are from $\{1,2,3,\dots,q\}$. A graph that admits skolem difference mean labeling is called a skolem difference mean graph.

Example 1.9: Skolem difference mean labeling of the cycle C_3 is given in figure 1.

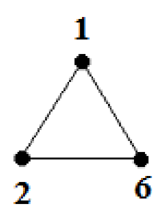

Figure 1

Result 1.9[8]: A necessary condition for a graph to be skolem difference mean is that $p \geq q$.

2. MAIN RESULTS

Definition 2.1: The edge reduced skolem difference mean number denoted by $\text{ER}\sigma_{sdm}(G)$ of a non skolem difference mean graph G is the minimum number of edges whose removal from G results in a skolem difference mean graph. If no such number exists then $\text{ER}\sigma_{sdm}(G) = \infty$.

Theorem 2.2: The graph obtained by removing the $(m-1)$ internal edges in $K_2 + mK_1$ is skolem difference mean. Thus ER$\sigma_{sdm}(K_2 + mK_1) = m - 1$.

Proof: Let G be the graph obtained by removing the $(m-1)$ internal edges of $K_2 + mK_1$.
Let $V(G) = \{u_i, v_j; 1 \leq i \leq 2, 1 \leq j \leq m\}$ and $E(G) = \{u_1 v_j; 1 \leq i \leq m\} \cup \{u_1 u_2, u_2 v_1\}$
Then $|V(G)| = |E(G)| = m + 2$. Let f: $V(G) \rightarrow \{1, 2, 3, \ldots, 2m + 4\}$ be defined as follows.
$f(v_1) = 2m + 4$
$f(v_i) = 2m + 4 - 2i; 2 \leq i \leq m$
$f(u_i) = 2i - 1; 1 \leq i \leq 2$
Let f^* be the induced edge labeling of f. Then
$f^*(u_1 u_2) = 1$
$f^*(u_1 v_1) = m + 2$
$f^*(u_2 v_1) = m + 1$
$f^*(u_1 v_i) = m + 2 - i; 2 \leq i \leq m$
The induced edge labels are distinct and are 1, 2, 3,…, m+2. Hence ER$\sigma_{sdm}(K_2 + mK_1) = m - 1$.

Example 2.3: Skolem difference mean labeling of the graph obtained by removing 4 internal edges of $K_2 + 5K_1$ is given in Figure 2.

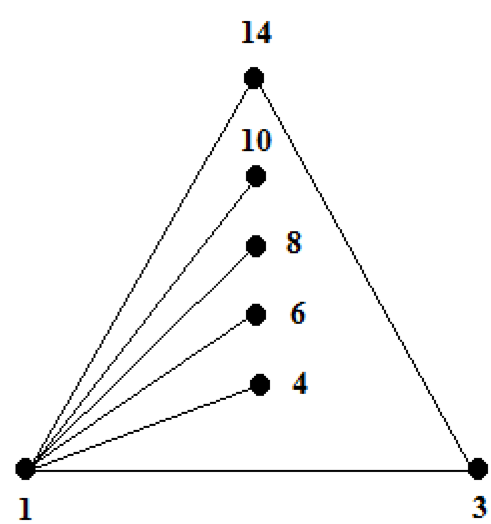

Figure 2

Theorem 2.4: Let G be the graph obtained by taking two copies of $K_{1,n}$ and joining the central vertex of the second copy with the pendant vertices of the first copy. Then $G - \{vu_i; 2 \leq i \leq n-1\}$ is skolem difference mean for all $n \geq 3$. The graph has edge reduced skolem difference mean number n−2.

Proof: Let $V(G - \{vu_i; 2 \leq i \leq n-1\}) = \{v, v_i, u, u_i; 1 \leq i \leq n\}$ and

$E(G - \{vu_i; 2 \leq i \leq n-1\}) = \{vv_i, uu_i, vu_1, vu_2; 1 \leq i \leq n\}$

Then $G - \{vu_i; 2 \leq i \leq n-1\}$ has $2n+2$ vertices and $2n+2$ edges.

Let f: $V(G - \{vu_i; 2 \leq i \leq n-1\}) \to \{1, 2, 3, \ldots, 4n+4\}$ be defined as follows.

$f(v) = 4n+4$

$f(v_1) = 1$

$f(v_n) = 3$

$f(v_i) = 2i+1; 2 \leq i \leq n-1$

$f(u) = 2n+5$

$f(u_i) = 4+2i; 1 \leq i \leq n$

Let f^* be the induced edge labeling of f. Then we have

$f^*(vv_1) = 2n+2$

$f^*(vv_n) = 2n+1$

$f^*(vv_i) = 2n+2-i; 2 \leq i \leq n-1$

$f^*(uv_1) = n+2$

$f^*(uv_n) = n+1$

$f^*(uu_i) = n+1-i; 1 \leq i \leq \mathrm{n}$

The induced edge labels are distinct and are $1, 2 \ldots 2n+2$. Hence the graph has edge reduced skolem difference mean number n−2.

Example 2.5: Skolem difference mean labeling of the graph obtained by taking two copies of K_{15} and joining the central vertex of the second copy with any two pendant vertices of the first copy is given in Figure 3.

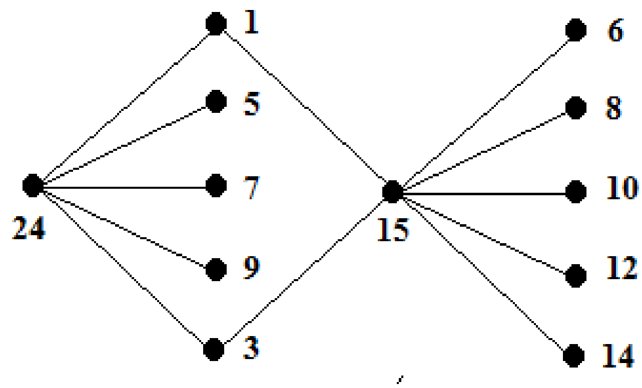

Figure 3

Theorem 2.6: $G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \le i \le n\}$ is skolem difference mean. Hence ER$\sigma_{sdm}\big(G_v(P_n)\big) = n - 4$.

Proof: Let $V(G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \le i \le n\}) = \{v_i; 1 \le i \le n\}$ and
$E(G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \le i \le n\}) = \{v_iv_{i+1,}v_1v_3, v_1v_4; 2 \le i \le n-1\}$
Then $|V(G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \le i \le n\})| = n$ and
$|E(G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \le i \le n\})| = n$
Let f: $V(G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \le i \le n\}) \rightarrow \{1, 2, 3 \dots, 2n\}$ be defined as follows.

Case $(\boldsymbol{i})$ when n is odd

$f(v_1) = 2n$

$f(v_{2i}) = 2n + 1 - 2i; 1 \le i \le \frac{n-1}{2}$

$f(v_{2i+1}) = 2i - 1;\ \ 1 \le i \le \frac{n-3}{2}$

$f(v_n) = f(v_{n-1}) - 2$

Case $(\boldsymbol{ii})$ when n is even

$f(v_1) = 2n$

$f(v_{2i}) = 2n + 1 - 2i; 1 \le i \le \frac{n-2}{2}$

$f(v_{2i+1}) = 2i - 1;\ \ 1 \le i \le \frac{n-2}{2}$

$f(v_n) = f(v_{n-1}) + 2$

In both the cases, let f^* be the induced edge labeling of f. Then

$f^*(v_1v_3) = n$

$f^*(v_1v_4) = 2$

$f^*(v_iv_{i+1}) = n + 1 - i; 2 \leq i \leq n - 2$

$f^*(v_{n-1}v_n) = 1$

The induced edge labels are distinct and are 1, 2, 3,…, n. Hence $G_v(P_n) - \{e_i \,/\, e_i = v_1v_i, 5 \leq i \leq n\}$ is skolem difference mean. Thus $ER\sigma_{sdm}\big(G_v(P_n)\big) = n - 4$.

Example 2.7: Skolem difference mean labeling of $G_v(P_8) - \{e_i \,/\, e_i = v_1v_i, 5 \leq i \leq n\}$ is given in figure 4.

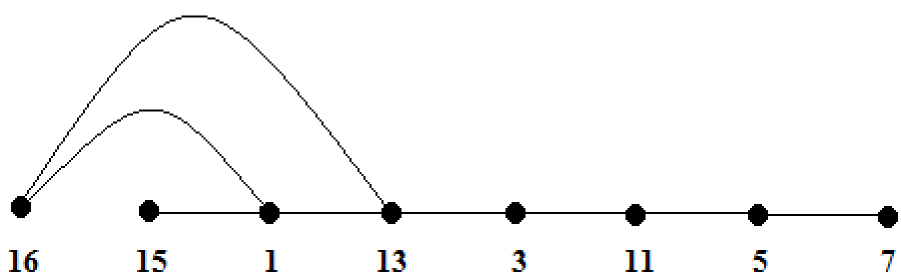

Figure 4

Theorem 2.8: $S(P_n) - \{e_i; e_i = u_iv_{i+1}, 3 \leq i \leq n\}$ is skolem difference mean. Hence $ER\sigma_{sdm}S(P_n) = n - 3$ for all $n \geq 4$.

Proof: Let $V(S(P_n) - \{e_i; e_i = u_iv_{i+1}, 3 \leq i \leq n\}) = \{v_i; 1 \leq i \leq n\} \cup \{u_j; 1 \leq j \leq n\}$ and $E\ \ (S(P_n) - \{e_i; e_i = u_iv_{i+1}, 3 \leq i \leq n\}) = \{v_iv_{i+1}; 1 \leq i \leq n-1\} \cup \{v_ju_{j+1}; 1 \leq j \leq n-1\} \cup \{u_1v_2, u_2v_3\}$

Then $S(P_n) - \{e_i; e_i = u_iv_{i+1}, 3 \leq i \leq n\}$ has $2n$ vertices and $2n$ edges.

Let f: $V(S(P_n) - \{e_i; e_i = u_iv_{i+1}, 3 \leq i \leq n\}) \rightarrow \{1, 2, 3, \dots, 4n\}$ be defined as follows.

Case $(\boldsymbol{i})$ when n is odd

$f(v_{2i+1}) = 1 + 2i; 0 \leq i < \frac{n+1}{2}$

$f(v_{2i}) = 4n + 2 - 2i; 1 \leq i < \frac{n+1}{2}$

$f(u_1) = 2n - 2$

$f(u_2) = 2n + 1$

$f(u_{2i+1}) = 2n + 2 + 2i; 1 \leq i < \frac{n+1}{2}$

$f(u_{2i}) = 2n + 1 - 2i; 2 \leq i < \frac{n+1}{2}$

Case $(\boldsymbol{ii})$ when *n* is even

$f(v_{2i+1}) = 1 + 2i; 0 \leq i < \frac{n}{2}$

$f(v_{2i}) = 4n + 2 - 2i; 1 \leq i \leq \frac{n}{2}$

$f(u_1) = 2n - 2$

$f(u_2) = 2n + 1$

$f(u_{2i+1}) = 2n + 2 + 2i; 1 \leq i < \frac{n}{2}$

$f(u_{2i}) = 2n + 1 - 2i; 2 \leq i \leq \frac{n}{2}$

In both the cases let f^* be the induced edge labeling of f. Then

$f^*(v_i v_{i+1}) = 2n + 1 - i; 1 \leq i \leq n - 1$

$f^*(v_1 u_2) = n$

$f^*(v_i u_{i+1}) = n - i; 2 \leq i \leq n - 1$

$f^*(u_1 v_2) = n + 1$

$f^*(u_2 v_3) = n - 1$

The induced edge labels are distinct and are $1,2,3,\ldots,2n$.Hence $S(P_n) - \{e_i; e_i = u_i v_{i+1}, 3 \leq i \leq n\}$ is skolem difference mean. Thus $ER\sigma_{sdm}S(P_n) = n - 3$ for all $n \geq 4$.

Example 2.9: Skolem difference mean labeling of $S(P_6) - \{e_i; e_i = u_i v_{i+1}, 3 \leq i \leq 6\}$ is given in Figure 5.

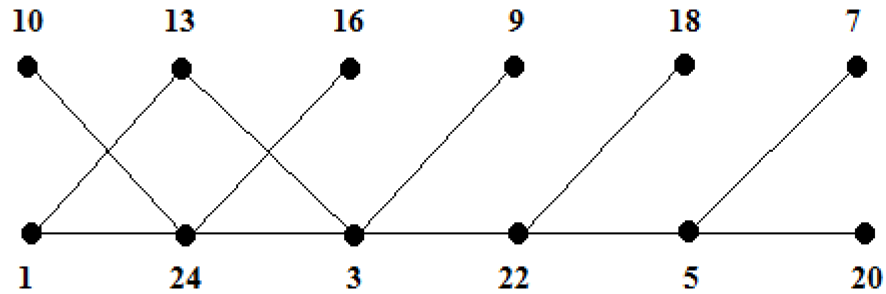

Figure 5

Theorem 2.10: $D_2(P_n) - \{v_i u_{i+1}, u_i v_{i+1}; 2 \leq i \leq n-1\}$is skolem difference mean. Hence $ER\sigma_{sdm}\big(D_2(P_n)\big) = n-2$ for all $n \geq 3$.

Proof: Let $V(D_2(P_n) - \{v_i u_{i+1}, u_i v_{i+1}; 2 \leq i \leq n-1\}) = \{v_i, u_i; 1 \leq i \leq n\}$ and
$E(D_2(P_n) - \{v_i u_{i+1}, u_i v_{i+1}; 2 \leq i \leq n-1\}) = \{v_i v_{i+1}, u_i u_{i+1}, v_1 u_2, u_1 v_2; 1 \leq i \leq n-1\}$
Let f: $V(D_2(P_n) - \{v_i u_{i+1}, u_i v_{i+1}; 2 \leq i \leq n-1\}) \to \{1, 2, 3, \dots, 4n\}$ be defined as follows.

Case$(\boldsymbol{i})$ when n is odd

$f(v_{2i+1}) = 2i+1; 0 \leq i < \frac{n+1}{2}$

$f(v_{2i}) = 4n+2-2i; 1 \leq i < \frac{n+1}{2}$

$f(u_{2i+1}) = 2n-2-2i; 0 \leq i < \frac{n+1}{2}$

$f(u_{2i}) = 2n-2+2i; 1 \leq i < \frac{n+1}{2}$

Case$(\boldsymbol{ii})$ when n is even

$f(v_{2i+1}) = 2i+1; 0 \leq i < \frac{n}{2}$

$f(v_{2i}) = 4n+2-2i; 1 \leq i \leq \frac{n}{2}$

$f(u_{2i+1}) = 2n-2-2i; 0 \leq i < \frac{n}{2}$

$f(u_{2i}) = 2n-2+2i; 1 \leq i \leq \frac{n}{2}$

In both cases, let f^* be the induced edge labeling of f. Then we have

$f^*(v_i v_{i+1}) = 2n+1-i; 1 \leq i \leq n-1$

$f^*(u_i u_{i+1}) = i; 1 \leq i \leq n-1$

$f^*(u_1 v_2) = n+1$

$f^*(v_1 u_2) = n$

The induced edge labels are distinct and are *1,2,3,…,2n.* Hence $D_2(P_n) - \{v_i u_{i+1}, u_i v_{i+1}; 2 \leq i \leq n-1\}$ is skolem difference mean. Thus $ER\sigma_{sdm}\big(D_2(P_n)\big) = n-2$ for all $n \geq 3$..

Example 2.10: Skolem difference mean labeling of $D_2(P_6) - \{v_i u_{i+1}, u_i v_{i+1}; 2 \leq i \leq 5\}$ is given in Figure 6.

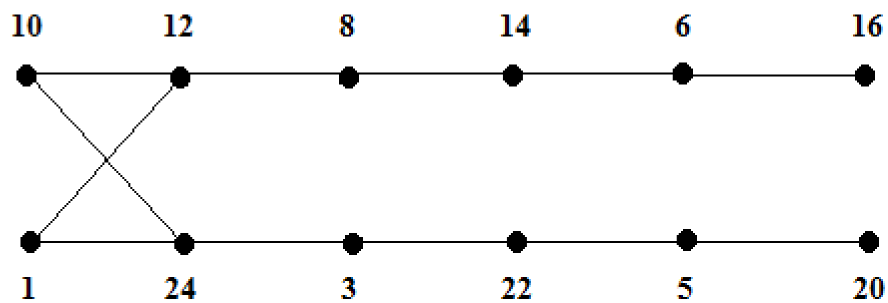

Figure 6

Theorem 2.11*:* L_{n+2}–$\{e_i; 4 \leq i \leq n+1\}$ where $e_i = v_i v_{i+1}$ is skolem difference mean. Thus ER$\sigma_{sdm}(L_{n+2})$ = n-2 for all $n \geq 3$.

Proof: Let $V(L_{n+2}$–$\{e_i; 4 \leq i \leq n+1\}) = \{u_i, v_i / 1 \leq i \leq n+2\}$ and

$E(L_{n+2}$–$\{e_i; 4 \leq i \leq n+1\}) = \{u_i u_{i+1}, v_i v_{i+1}, u_j v_j; 1 \leq i \leq n+1, 2 \leq j \leq 3\}$

Then L_{n+2}–$\{e_i; 4 \leq i \leq n+1\}$ has $2n+4$ vertices and $2n$+4 edges.

Let *f*: $V(L_{n+2}$–$\{e_i; 4 \leq i \leq n+1\}) \rightarrow \{1, 2 \ldots 4n+8\}$ be defined as follows.

Case$(\boldsymbol{i})$ when n is odd.

Sub case$(\boldsymbol{a})$: when $n = 4m+1$ where $m \geq 1$

$f(u_{2i+1}) = 2i+1; 0 \leq i \leq \frac{n+1}{2}$

$f(u_{2i}) = 4n+10-2i; 1 \leq i \leq \frac{n+1}{2}$

$f(v_{2i+1}) = n + 4 + 2i; 0 \leq i \leq \frac{n+3}{4}$

$= n + 6 + 2i; \frac{n+3}{4} < i \leq \frac{n+1}{2}$

$f(v_{2i}) = 3n + 12 - 2i; 1 \leq i \leq \frac{n+3}{4}$

$= 3n + 10 - 2i; \frac{n+3}{4} < i \leq \frac{n+1}{2}$

Sub case$(\boldsymbol{b})$: when $n = 4m + 3$ where $m \geq 1$

$f(u_{2i+1}) = 2i + 1; 0 \leq i \leq \frac{n+1}{2}$

$f(u_{2i}) = 4n + 10 - 2i; 1 \leq i \leq \frac{n+1}{2}$

$f(v_{2i+1}) = n + 4 + 2i; 0 \leq i \leq \frac{n+1}{4}$

$= n + 6 + 2i; \frac{n+1}{4} < i \leq \frac{n+1}{2}$

$f(v_{2i}) = 3n + 12 - 2i; 1 \leq i \leq \frac{n+5}{4}$

$= 3n + 10 - 2i; \frac{n+5}{4} < i \leq \frac{n+1}{2}$

Case$(\boldsymbol{ii})$ when n is even.

Sub case$(\boldsymbol{a})$: when $n = 4m$ where $m \geq 1$

$f(u_{2i+1}) = 2i + 1; 0 \leq i \leq \frac{n}{2}$

$f(u_{2i}) = 4n + 10 - 2i; 1 \leq i \leq \frac{n+2}{2}$

$f(v_{2i+1}) = n + 3 + 2i; 0 \leq i \leq \frac{n}{2}$

$f(v_{2i}) = 3n + 11 - 2i; 1 \leq i \leq \frac{n+4}{4}$

$$= 3n + 7 - 2i; \frac{n+4}{4} < i \leq \frac{n+2}{2}$$

Sub case$(\boldsymbol{b})$: when $n = 4m + 2$ where $m \geq 1$

$f(u_{2i+1}) = 2i + 1; 0 \leq i \leq \frac{n}{2}$

$f(u_{2i}) = 4n + 10 - 2i; 1 \leq i \leq \frac{n+2}{2}$

$f(v_{2i+1}) = n + 3 + 2i; 0 \leq i \leq \frac{n+2}{4}$

$$= n + 7 + 2i; \frac{n+2}{4} < i \leq \frac{n}{2}$$

$f(v_{2i}) = 3n + 11 - 2i; 1 \leq i \leq \frac{n+2}{2}$

In both the case, let f^* be the induced edge labeling of f. Then we have

Case($\boldsymbol{i}$) when n is odd

$f^*(u_i u_{i+1}) = 2n + 5 - i; 1 \leq i \leq n + 1$

$f^*(u_2 v_2) = \frac{n-1}{2}$

$f^*(u_3 v_3) = \frac{n+3}{2}$

$f^*(v_i v_{i+1}) = n + 4 - i; 1 \leq i \leq \frac{n+3}{2}$

$= \frac{n+1}{2}$ when i $= \frac{n+5}{2}$

$$= n + 2 - i; \frac{n+5}{2} < i \leq n + 1$$

Case($\boldsymbol{ii}$) when n is even

$f^*(u_i u_{i+1}) = 2n + 5 - i; 1 \leq i \leq n + 1$

$f^*(u_2 v_2) = \frac{n}{2}$

$f^*(u_3 v_3) = \frac{n+2}{2}$

$f^*(v_i v_{i+1}) = n + 4 - i; 1 \leq i \leq \frac{n+4}{2}$

$$= n + 2 - i; \frac{n+4}{2} < i \leq n + 1$$

The induced edge labels are distinct and are $1,2,3,\ldots,2n+4$. Hence $L_{n+2-}\{e_i; 4 \leq i \leq n + 1\}$ where $e_i = v_i v_{i+1}$ is skolem difference mean for all $n \geq 3$.Hence edge reduced skolem difference mean number of L_{n+2} is n-2.

Example 2.13: Skolem difference mean labeling of $L_{10-}\{e_i; 4 \leq i \leq 9\}$ where $e_i = v_i v_{i+1}$ is given in figure 7.

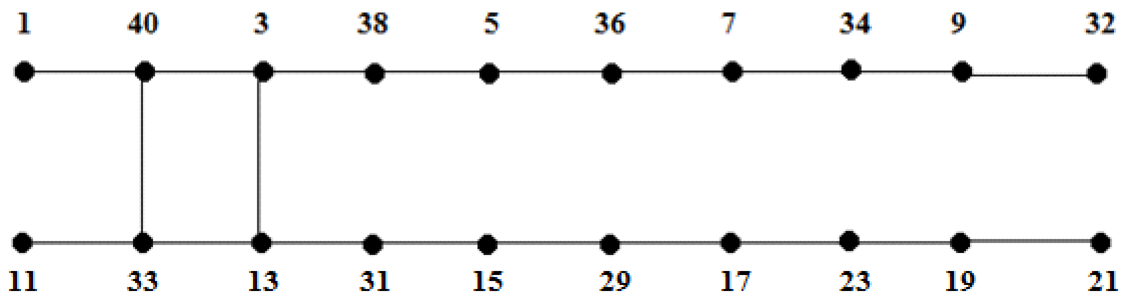

Figure 7

3. CONCLUSION

In this paper, the author studied the edge reduced skolem difference mean number of some graphs. Further studies can be made for graphs G for which $ER\sigma_{sdm}(G) > q - p$.

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Dr. K. Murugan is an Assistant Professor of Mathematics in The M.D.T Hindu College, Tirunelveli. He has been awarded Ph.D degree in Mathematics by the Manonmaniam Sundaranar University, Tirunelveli in 2013. The title of his thesis is 'STUDIES IN GRAPH THEORY-SKOLEM DIFFERENCE MEAN LABELING AND RELATED TOPICS'. His thrust area is Graph labeling. He has participated in more than 50 conferences, presented 23 research papers, given seven guest lectures and served as a resource person in a seminar. One of his research papers has been awarded 'BEST RESEARCH PAPER-MATHEMATICS'

References

[1] B. D. Acharya, Construction of certain infinite families of graceful graphs from a given graceful graph, *Def. Sci. J.* 32(3) (1982) 231-236.

[2] B. D. Acharya, S. B. Rao and S. Arumugam, Embedding and NP-complete problems for graceful graphs, Proc. Labelings of discrete structures and applications, (Eds: B.D. Acharya, S. Arumugam, A. Rosa) Narosa Publishing House, 2008, 57-62.

[3] Frank Harary, Graph Theory, Narosa Publishing House, New Delhi, 2001.

[4] Joseph A. Gallian, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics,* 15(2008), #DS6.

[5] K. Murugan and A. Subramanian, Labeling of Sub divided graphs, *American Journal of Mathematics and Sciences*, 1(1) (2012) 143-149.

[6] K. Murugan and A. Subramanian, Skolem difference mean graphs, *Mapana Journal of Sciences*, 11(4) (2012) 109-120.

[7] K. Murugan and A. Subramanian, Skolem difference mean labeling of H-graphs, *International Journal of Mathematics and Soft Computing,* 1(1) (2011)115-129.

[8] K. Murugan, Some results on skolem difference mean graphs, *International Journal of Mathematics and its Applications*, 3(3D) (2015) 75-80.

[9] D. Ramya, M. Selvi and R. Kalaiyarasi, On skolem difference mean labelling of graphs, *International Journal of Mathematical Archive*, 4(12) (2013) 73-79.

[10] A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (International symposium, Rome, July1966), Gorden and Breach, N.Y and Dunod Paris, 1967, 349-355.

[11] Sampathkumar E. and Walikar H. G, On the splitting graph of a graph, *The Karnataka University Journal Science,* Vol. XXX & XXXI (combined) (1980-1981).

[12] M.Selvi and D.Ramya, On skolem difference mean labelling of some trees, *International Journal of Mathematics and Soft Computing,* 4(2) 2014) 11-18.

[13] M. Selvi, D. Ramya and P. Jeyanthi, Skolem difference mean graphs, *Proyecciones Journal of Mathematics,* 34(3) (2015) 243-254.

[14] S. K. Vaidya and N. H. Shah, Some new odd harmonious graphs, *International Journal of Mathematics and Soft Computing,* 1(1) (2011) 9-11.

[15] S. K. Vaidya and P. L. Vihol, Fibonacci and Super Fibonacci graceful labeling of some graphs, *Studies in Mathematical Sciences*, 2(2) (2011) 24-35.

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