Available online at www.worldscientificnews.com

World Scientific News

WSN 13 (2015) 101-115 EISSN 2392-2192

A semiclassical model for a laser field inside an optical cavity

K. M. Priyashanka, K. A. I. L. Wijewardena Gamalath*

Department of Physics, University of Colombo, Colombo3, Sri Lanka

*E-mail address: imalie@phys.cmb.ac.lk

ABSTRACT

Treating the laser medium quantum mechanically and the laser electric field classically, a mathematical model was developed to study a laser field inside a single mode optical cavity by numerical and analytical techniques. The simulations for threshold population, population inversion and average population with electric field frequency for 500 kHz resonance frequency of the atom are presented. The gain coefficient and the pump parameter with the frequency of the cavity electric field for 1000 kHz resonance frequency of the atom were simulated. The threshold population is a minimum when the frequency of electric field is equal to the resonance frequency of the atom. The population inversion varies sinusoidal with time and the frequency of the electric field and is a maximum at the threshold frequency of the atom. The saturated laser intensity at steady state increases linearly with the pump parameter and for larger values of the pump parameter, a smaller time is needed to reach the saturation of the laser field.

***Keywords*:** Semi classical theory; cavity electric field; population inversion; pump parameter; resonance frequency; threshold population

1. INTRODUCTION

In 1917 Einstein established the foundation of the lasers by suggesting the existence of the stimulated emission phenomena [1]. This phenomenon was applied for the amplification of the electromagnetic field and to combine the amplifier with a resonator to make an oscillator [2]. Laser radiation is produced in a resonant cavity where the resonant frequency of the cavity is the same as the frequency associated with the atomic electron transitions providing energy flow into the field. As energy in the resonant mode builds up, the probability for stimulated emission only in that mode increases. In a laser, the light emitted in to this resonant state is highly coherent [3]. The semi-classical theory ignores the quantum mechanical nature of the electromagnetic field while the Schrodinger equation is used to determine the behaviour of the matter. Though this theory does not recognize the affect on atomic behaviour associated with quantum fluctuations of the electromagnetic field except in isolated instances, the justification for a semiclassical theory is extremely strong in the domain embracing all of radiation theory where number of photons is much larger than unity. In most lasers, the number of photons is unbounded and the effects due to filed quantization are insignificant so long as the attention remains on the stimulated process [1]. A major strength of the semiclassical theory is that it gives a quantitative picture of the interaction between laser modes. Introducing the laser field in self-consisting manner, Lamb [4] obtained the equation of motion for the field amplitude in semiclassical theory. The variation in the electric field intensity transverse to the laser axis is slowly varying compared to the optical wavelengths and can be neglected [5]. Spontaneous emissions do not appear naturally in semiclassical theories, but with the introduction of an additive noise to the model it can be modified to a Langevin type equation which can be simplified to a Fokker-Planck equation similar to the quantum mechanical treatment which automatically include spontaneous emissions.

In the present work, a simple semi classical mathematical model is developed to study the properties and behaviour of an optical cavity laser with a medium of two level atoms operating in a single mode. Treating the electric field in the optical cavity classically while the energy states of atoms to be quantized, a simple mathematical model for population inversion of the atoms in laser medium was obtained and the master equation of the density operator of the laser field was studied analytically and numerically. The Fokker-Plank equation for the phase space density for the laser field was solved analytically for both time dependent and steady state situations using coherent states. The simulations are presented at a $500\,\mathrm{kHz}$ resonance frequency for threshold population, population inversion and average population with electric field frequency. The gain and the pump parameter at a 1000 kHz resonance frequency with electric field frequency were also simulated.

2. SEMI-CLASSICAL APPROACH

The Maxwell-Bloch equation provides the basis of the theory of laser operation. In this theory the atomic dipoles determined by the off diagonal density matrix elements serves as sources of radiation. This radiation in turn drives these dipoles into oscillations causing them to radiate. The atoms in the medium were considered to have two quantized energy states with an energy separation $\hbar\omega_0$ and the electric field $\boldsymbol{E}$ present in the laser cavity has an angular frequency ω. The field acting on each excited atom will introduces a dipole moment, and the sum of those dipole moments will constitute macroscopic polarization $\boldsymbol{P}$ in the laser medium. This polarization acting as a source in Maxwell equations gives rise to a reaction laser field $\boldsymbol{E}'$. Due to self-consistency the reaction laser field is equal to electric field in the laser cavity. For electromagnetic waves in a macroscopic, homogeneous, isotropic dielectric material with charge density ρ and current density $\boldsymbol{J}$, the Maxwell's equations for electric field $\boldsymbol{E}$ and magnetic field strength $\boldsymbol{H}$ in SI units are given by

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \quad \nabla\times\boldsymbol{H} = \boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t}, \quad \nabla\cdot\boldsymbol{D} = 0 = \nabla\cdot\boldsymbol{B} \qquad (1)$$

where $\boldsymbol{D} = \varepsilon_0\boldsymbol{E}+\boldsymbol{P}$, $\boldsymbol{B} = \mu_0\boldsymbol{H}$ and $\boldsymbol{J} = \sigma\boldsymbol{E}$. Equation 1 leads to:

$$\nabla\times\nabla\times\boldsymbol{E} = -\mu_0\frac{\partial\left(\nabla\times\boldsymbol{H}\right)}{\partial t} = -\mu_0\frac{\partial}{\partial t}\left(\sigma\boldsymbol{E}+\frac{\partial}{\partial t}\left(\varepsilon_0\boldsymbol{E}+\boldsymbol{P}\right)\right) = -\mu_0\left(\sigma\dot{\boldsymbol{E}}+\varepsilon_0\ddot{\boldsymbol{E}}+\ddot{\boldsymbol{P}}\right) \qquad (2)$$

The left hand side of the above equation can be written using the vector identity $\nabla\left(\nabla\cdot\boldsymbol{E}\right)-\nabla^2\boldsymbol{E}$ and the principle spatial variation of $\boldsymbol{E}$ in the direction of the laser axis taken as z axis is obtained by neglecting x, y derivatives as,

$$\frac{\partial^2 E}{\partial z^2} - \mu_0\sigma\dot{E} - \frac{\ddot{E}}{c^2} = \mu_0\ddot{P} \qquad (3)$$

The time dependence of the above wave equation can be separated from the spatial dependence by the expansion of the electric field in normal modes of the cavity. Then the electric field can be written as

$$E\left(r,t\right) = \sum_n A_n(t)U_n\left(r\right)+c.c\ldots \qquad (4)$$

and the complex electric field amplitude of the n^{th} mode $A_n(t)$ is taken as

$$A_n\left(t\right) = \varepsilon_n\left(t\right)e^{-i\left(\omega_n t+\varphi_n(t)\right)} \qquad (5)$$

where the amplitude coefficient $\varepsilon_n\left(t\right)$ and phase $\varphi_n(t)$ are slowly varying functions of time in an optical frequency period and $\omega_n+\dot{\varphi}_n$ is the oscillation frequency of the mode. In the case of an optical cavity formed by plane mirrors at $z=0$ and $z=L$, $U\left(r\right)=\exp\left(ikz\right)$ with $k=n\pi c/L$. For a laser with only a single cavity mode excited, equation 4 read:

$$E\left(r,t\right) = \varepsilon\left(t\right)e^{-i\left(\omega t+\varphi(t)\right)}U\left(r\right)+c.c\ldots \qquad (6)$$

The polarization of medium having the same frequency, can be written as

$$P(r,t) \;=\; p(t)e^{-i(\omega t+\varphi(t))}U(r)+c.c\ldots \qquad (7)$$

where $p(t)$ is a slowly varying function of time. For slowly varying functions

$$e|\ddot{\varepsilon}(t)| \ll \omega|\dot{\varepsilon}(t)| \ll \omega^2|\varepsilon(t)| \qquad (8)$$

Substituting above results in equation 3 and neglecting $\ddot{\varepsilon},\ \ddot{\varphi},\ \ddot{p},\ \dot{\varepsilon},\ \dot{\varphi},\ \dot{p},\ \sigma\dot{\varepsilon},\ \sigma\dot{\varphi},\ \dot{\varphi}p$:

$$2\omega\left(ck-\omega-\dot{\varphi}(t)\right)\varepsilon(t)-i\mu_0\sigma\omega c^2\varepsilon(t)-2i\omega\dot{\varepsilon}(t) \;=\; \mu_0\omega^2c^2p(t) \qquad (9)$$

By comparing real and imaginary parts we obtain

$$\dot{\varepsilon}(t)+\frac{1}{2}c^2\mu_0\sigma\varepsilon(t) \;=\; -\frac{1}{2}c^2\mu_0\omega\,\mathrm{Im}\left(p(t)\right) \qquad (10)$$

$$\dot{\varphi}(t)\varepsilon(t)-\left(ck-\omega\right)\varepsilon(t) \;=\; -\frac{1}{2}c^2\mu_0\omega^2\,\mathrm{Re}\left(p(t)\right) \qquad (11)$$

For a two level atom with energy levels E_1 and E_2 with energy difference $E_2-E_1=\hbar\omega_0$, the wave functions for the energy states are given by $|\psi_i\rangle=|\phi_i\rangle e^{-iE_it/\hbar}$ $(i=1,2)$. When an atom is interacting with radiation field due to the electric dipole operator $\boldsymbol{d}=e\boldsymbol{r},$ the free atom Hamiltonian H_0 modifies as

$$H=H_0+H_d\ ,\quad H_d=-\boldsymbol{d}\cdot\boldsymbol{E}(\boldsymbol{r},t). \qquad (12)$$

The total wave function can be written as a linear combination of the two energy states

$|\psi_1\rangle=|\phi_1\rangle e^{-iE_1t/\hbar}$ and $|\psi_2\rangle=|\phi_2\rangle e^{-iE_2t/\hbar}$,

$$|\psi\rangle \;=\; C_1(t)|\psi_1\rangle+C_2(t)|\psi_2\rangle \qquad (13)$$

From the Schrodinger equation $H_0|\psi_i\rangle=E_i|\psi_i\rangle$ and equation11 reads:

$$\begin{aligned} C_1E_1|\psi_1\rangle+C_2E_2|\psi_2\rangle+H_d|\psi\rangle \;&=\; i\hbar\frac{\partial}{\partial t}\left(C_1(t)|\psi_1\rangle+C_2(t)|\psi_2\rangle\right) \\ H_d|\psi\rangle \;&=\; i\hbar\left(\dot{C}_1|\psi_1\rangle+\dot{C}_2|\psi_2\rangle\right) \end{aligned} \qquad (14)$$

Since $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthonormal, taking the inner product with $|\psi_1\rangle$ and $|\psi_2\rangle$ leads to:

$$\begin{aligned} i\hbar\dot{C}_1 &= \langle\psi_1|H_d|\psi\rangle = -\boldsymbol{E}(\boldsymbol{r},t)\langle\psi_1|\boldsymbol{d}|\psi\rangle, \\ i\hbar\dot{C}_2 &= \langle\psi_2|H_d|\psi\rangle = -\boldsymbol{E}(\boldsymbol{r},t)\langle\psi_2|\boldsymbol{d}|\psi\rangle \end{aligned} \qquad (15)$$

Since all atomic states have well defined parity i.e., $\langle\psi_n|\boldsymbol{d}|\psi_m\rangle = 0 \ (n = m)$. Therefore

$$i\hbar\dot{C}_1 = -\boldsymbol{E}(\boldsymbol{r},t)C_2\langle\phi_1|\boldsymbol{d}|\phi_2\rangle e^{-it(E_2-E_1)/\hbar} = -\boldsymbol{E}(\boldsymbol{r},t)C_2D_{12}e^{-i\theta t} \qquad (16)$$

where $D_{12} = \langle\phi_1|\boldsymbol{d}|\phi_2\rangle$. Also the exponential decay of the excited states are considered and the energy difference term is taken to be complex $(E_2 - E_1)/\hbar = \omega_0 + i\gamma/2 = \theta$ where γ is the decay constant. Similarly,

$$\dot{C}_2 = \frac{i}{\hbar}\boldsymbol{E}(\boldsymbol{r},t)C_1D_{21}e^{i\theta t} \qquad (17)$$

where $D_{21} = D_{12}^*$. The dipole moment induced by the incident radiation is given by

$$D(t) = \langle\psi|\boldsymbol{d}|\psi\rangle = C_1^*C_2D_{12}e^{-i\theta t} + C_2^*C_1D_{21}e^{i\theta t} \qquad (18)$$

The sum of these dipole moments over the laser cavity will give the macroscopic polarization P. If $N(\boldsymbol{r})$ is the atomic density, then polarization given by,

$$P(\boldsymbol{r},t) = N(\boldsymbol{r})D(t) = N(\boldsymbol{r})(C_1^*C_2D_{12}e^{-i\theta t} + C_2^*C_1D_{21}e^{i\theta t}) = N(\boldsymbol{r})C_1^*C_2D_{12}e^{-i\theta t} + cc \qquad (19)$$

Differentiating above equation with respect to time

$$\dot{P}(\boldsymbol{r},t) = N(\boldsymbol{r})\left((\dot{C}_1^*C_2 + C_1^*\dot{C}_2 - i\theta C_1^*C_2)e^{-i\theta t}D_{12} + cc\right) \qquad (20)$$

and substituting for $\dot{C}_1$ and $\dot{C}_2$ from equation 16 and 17

$$\dot{P}(\boldsymbol{r},t) = -\frac{i}{\hbar}N(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r},t)|D_{12}|^2\rho(t) - i\theta P(\boldsymbol{r},t) \qquad (21)$$

where $\rho(t) = |C_1|^2 - |C_2|^2$ is the population inversion. Using the polarization given in equation 7,

$$\dot{P}(r,t) = (\dot{p}(t) - i(\omega + \dot{\varphi})p(t))e^{-i(\omega t + \varphi(t))}U(r) \cong -i\omega P(r,t) \tag{22}$$

leads to

$$P(r,t) = \frac{N(\boldsymbol{r})|D_{12}|^2 \rho(t)}{\hbar(\omega - \theta)} \boldsymbol{E}(\boldsymbol{r},t) \tag{23}$$

and electric field given in equation 6 in equation 23:

$$p(t) = \frac{N(\boldsymbol{r})|D_{12}|^2 \rho(t)\varepsilon(t)}{\hbar((\omega-\omega_0)^2 + \gamma^2/4)}(\omega - \omega_0 - i\gamma/2) \qquad (\theta = \omega_0 + i\gamma/2) \tag{24}$$

Substituting equation 24 in equations 10 and 11 the equations of motion for a laser field are obtained as

$$\dot{\varepsilon}(t) + \frac{1}{2}c^2\mu_0\sigma\varepsilon(t) = \frac{\gamma c^2 \mu_0 \omega}{4\hbar}\frac{N_{(r)}|D_{12}|^2 \rho(t)\varepsilon(t)}{((\omega-\omega_0)^2 + \gamma^2/4)} \tag{25}$$

$$\dot{\varphi}(t) = -\frac{c^2\mu_0\omega^2(\omega-\omega_0)}{2\hbar}\frac{N_{(r)}|D_{12}|^2\rho(t)}{((\omega-\omega_0)^2+\gamma^2/4)} + (ck - \omega) \tag{26}$$

Equation 25 describes the exponential growth or exponential decay when $\dot{\varepsilon}(t)$ is positive or negative respectively, i.e.

$$\frac{\gamma\omega}{4\hbar}\frac{N(\boldsymbol{r})|D_{12}|^2\rho(t)}{((\omega-\omega_0)^2+\gamma^2/4)} \begin{array}{ll} > \sigma/2 & (\dot{\varepsilon}(t) > 0) \\ < \sigma/2 & (\dot{\varepsilon}(t) < 0) \end{array} \tag{27}$$

The left hand side of the equation giving the population inversion $\rho(t)$ represents the gain of the system while the term in the right which is proportional to conductivity σ represents the loss. When equality holds, the gain of the system equalling to the loss, the system is at the threshold. Therefore threshold condition requires the minimum population inversion

$$N(\boldsymbol{r})\rho(t) = \frac{2\sigma\hbar((\omega-\omega_0)^2+\gamma^2/4)}{\gamma\omega|D_{12}|^2} \tag{28}$$

Above the threshold, when the gain exceed the loss, the amplitudes of $\varepsilon(t)$ grows exponentially. By simple algebraic procedure an equation for population inversion $\rho(t)$ can be obtained. Initially if $C_1(0)=1$ and $C_2(0)=0$, then all the electrons are in the lowest state and the population of the excited state is zero. Differentiating the equation17 with respect to time,

$$\ddot{C}_2 \;=\; \frac{i}{\hbar} D_{21} e^{i\theta t}\left(\boldsymbol{E}(\boldsymbol{r},t)\dot{C}_1 + i\theta \boldsymbol{E}(\boldsymbol{r},t)C_1 + \dot{\boldsymbol{E}}(\boldsymbol{r},t)C_1\right) \qquad (29)$$

Time differentiation of the equation 6 gives

$$\dot{\boldsymbol{E}}(\boldsymbol{r},t) \;=\; \left(\dot{\varepsilon}(t) - i(\omega+\dot{\varphi})\varepsilon(t)\right) e^{-i(\omega t+\varphi(t))} U(r) \;\cong\; -i\omega \boldsymbol{E}(\boldsymbol{r},t) \qquad (30)$$

Substituting the coefficients of C_1 and $\dot{C}_1$ from equation 16 and 17 in equation 29,

$$\ddot{C}_2 \; - i(\theta-\omega)\dot{C}_2 + \frac{1}{\hbar^2}\left|\boldsymbol{E}(\boldsymbol{r},t)\right|^2 \left|D_{12}\right|^2 C_2 \;=\; 0 . \qquad (31)$$

For the trial solution $C_2 = e^{i\mu t}$ the above equation read

$$\mu^2 - (\theta-\omega)\,\mu - \frac{1}{\hbar^2}\left|E(\boldsymbol{r},t)\right|^2 \left|D_{12}\right|^2 \;=\; 0 \qquad (32)$$

has the general solution

$$C_2(t) \;=\; A_+ e^{i\mu_+ t} + A_- e^{i\mu_- t} \qquad \left(\mu_\pm = \frac{1}{2}(\theta-\omega) \pm \frac{1}{2}\sqrt{(\theta-\omega)^2 + \frac{4}{\hbar^2}\left|E_{(r,t)}\right|^2 \left|D_{12}\right|^2}\right) \qquad (33)$$

For the initial condition when all the electrons are in the lowest state $A_+ = -A_- = A(say)$ and second equation of equation 33 give

$$\dot{C}_2(0) \;=\frac{i}{\hbar} E(\boldsymbol{r},0) D_{21} \;=\; iA\left(\mu_+ - \mu_-\right) \;\Rightarrow\; A \;=\; \frac{E(\boldsymbol{r},0)D_{21}}{\hbar\left(\mu_+ - \mu_-\right)} \qquad (34)$$

If $a = \frac{1}{2}(\theta-\omega)$ and $b = \frac{1}{2}\sqrt{(\theta-\omega)^2 + \frac{4}{\hbar^2}\left|E_{(r,t)}\right|^2 \left|D_{12}\right|^2}$, then the general solution:

$$C_2(t) \;=\; \frac{i}{\hbar} E(\boldsymbol{r},0) D_{21} e^{iat}\left(\frac{\sin bt}{b}\right) \qquad (35)$$

$$|C_2|^2 \;=\; \frac{|E(\boldsymbol{r},0)|^2 |D_{21}|^2}{b^2\hbar^2}\sin^2(bt) \qquad (36)$$

Similarly the solution for C_1 with initial conditions gives $A_- = \mu_+ / (\mu_+ - \mu_-)$ and $A_+ = -\mu_- / (\mu_+ - \mu_-)$

$$C_1 = \frac{-\mu_- e^{i\mu_+ t}}{\mu_+ - \mu_-} + \frac{\mu_+ e^{i\mu_- t}}{\mu_+ - \mu_-} = \frac{e^{iat}}{b}\left[-ia \sin bt + b \cos bt\right] \qquad (37)$$

and

$$|C_1|^2 = \frac{1}{b^2}\left[a^2 \sin^2 bt + b^2 \cos^2 bt\right] \qquad (38)$$

Then population inversion is given by

$$\rho(t) = |C_2|^2 - |C_1|^2 = \left(\frac{|E(\mathbf{r},0)|^2 |D_{21}|^2}{\hbar^2} - a^2\right)\frac{1}{b^2}\sin^2 bt - \cos^2 bt \qquad (39)$$

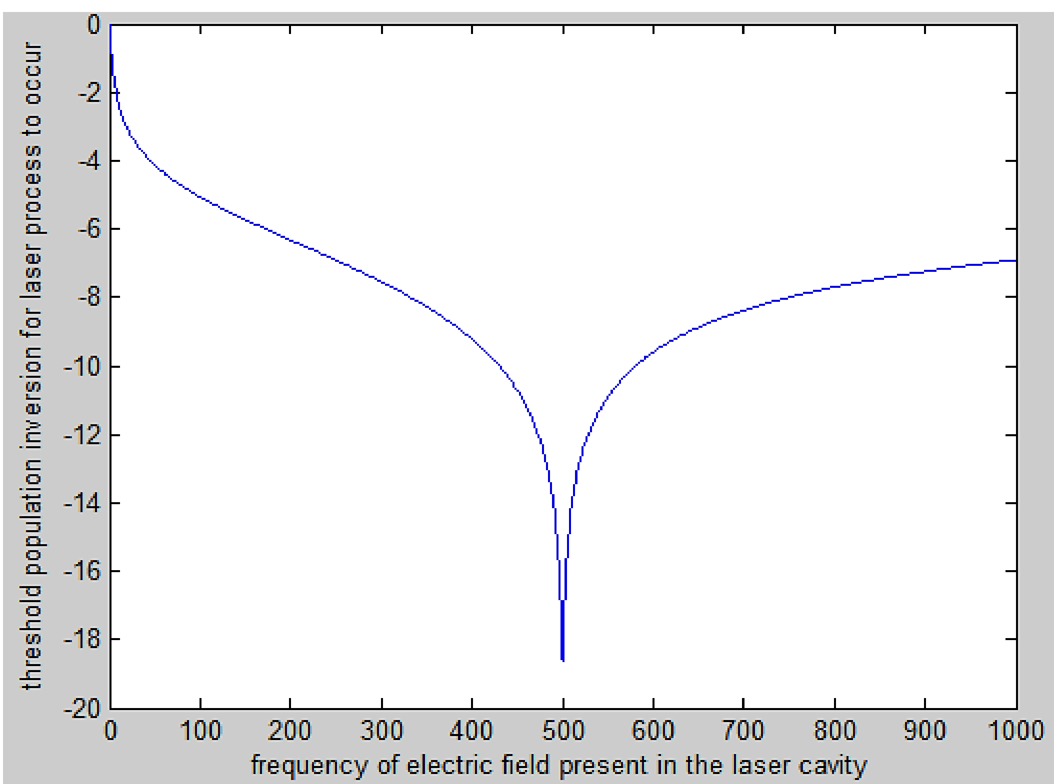

Figure 1. Log of threshold population inversion with the frequency of laser field inside the cavity. The resonance frequency of the atom ($\omega_0 = 500$ kHz)

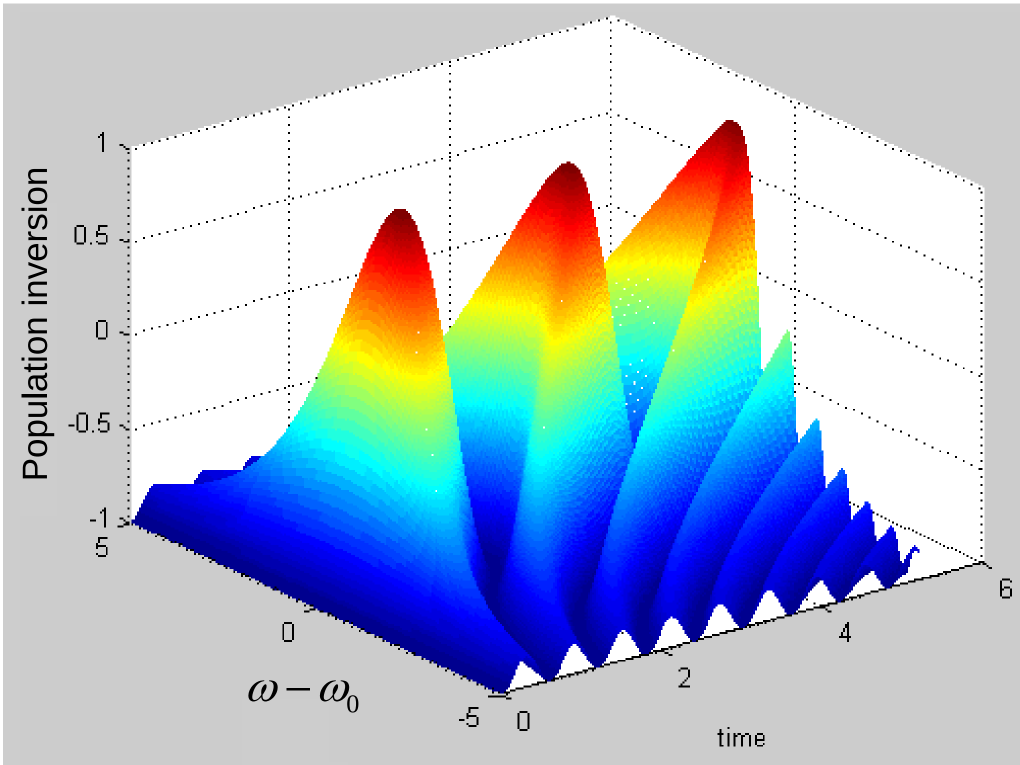

Figure 2. Population inversion $\rho(t)$ with time t and the difference between electric field frequency and resonance frequency of the atom $\omega - \omega_0$

Taking average over time:

$$\rho(t)_{avg} \cong 1 - \left(\frac{2|\varepsilon(t)||D_{12}|\left((\omega-\omega_0)-\frac{i\gamma}{2}\right)}{\hbar\left((\omega-\omega_0)^2+\frac{\gamma^2}{4}\right)} \right). \qquad (40)$$

The threshold population inversion which is necessary for laser action to occur is shown in Figure 1. A log scale was used for the y axis and the resonance frequency of two laser levels was taken as 500 kHz, a value that depends on the medium. The figure shows that the required threshold population is a minimum when the frequency of the electric field equals to the resonance frequency of atom. The threshold value increases rapidly near this minimum and the rate of increasing, decreases dramatically when going further and further away from that minimum.

The population inversion with time and with respect to the difference of the electric field frequency and resonance frequency of the atom $\omega - \omega_0$ is presented in the Figure 2. The population inversion vary between -1 and $+1$, as extreme cases are when all the electrons are either in the lower energy level or in the excited energy level.

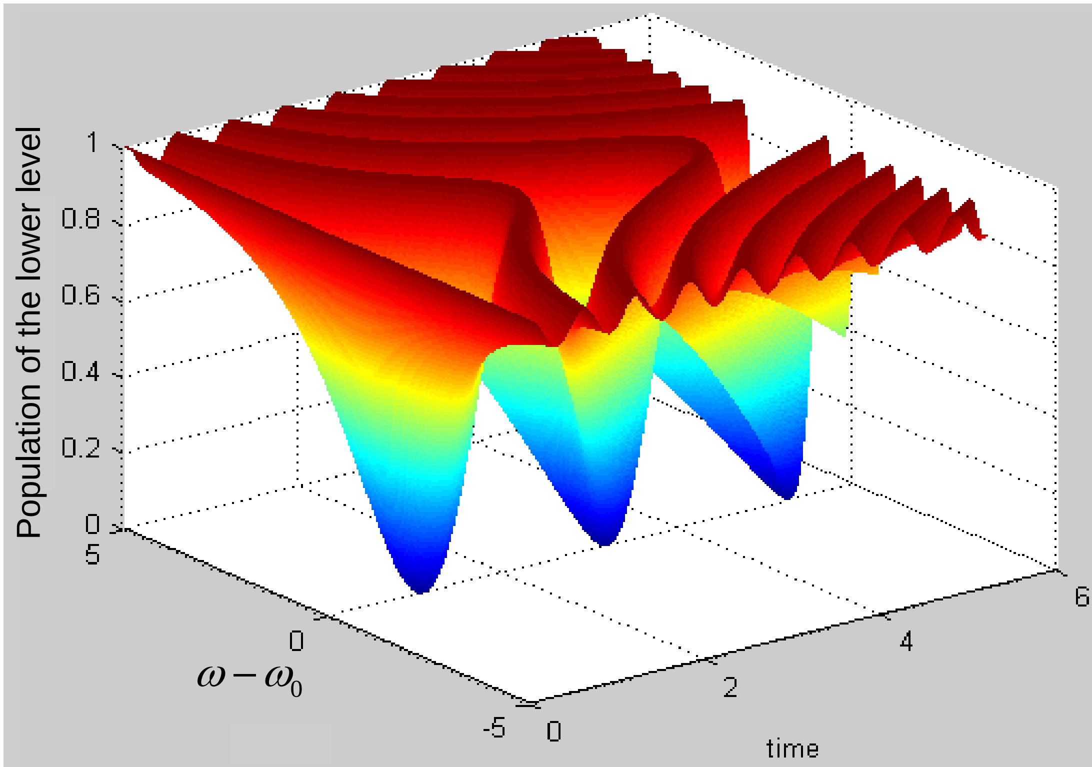

Figure 3. Population of the lower energy state as a function of $\omega - \omega_0$ and time

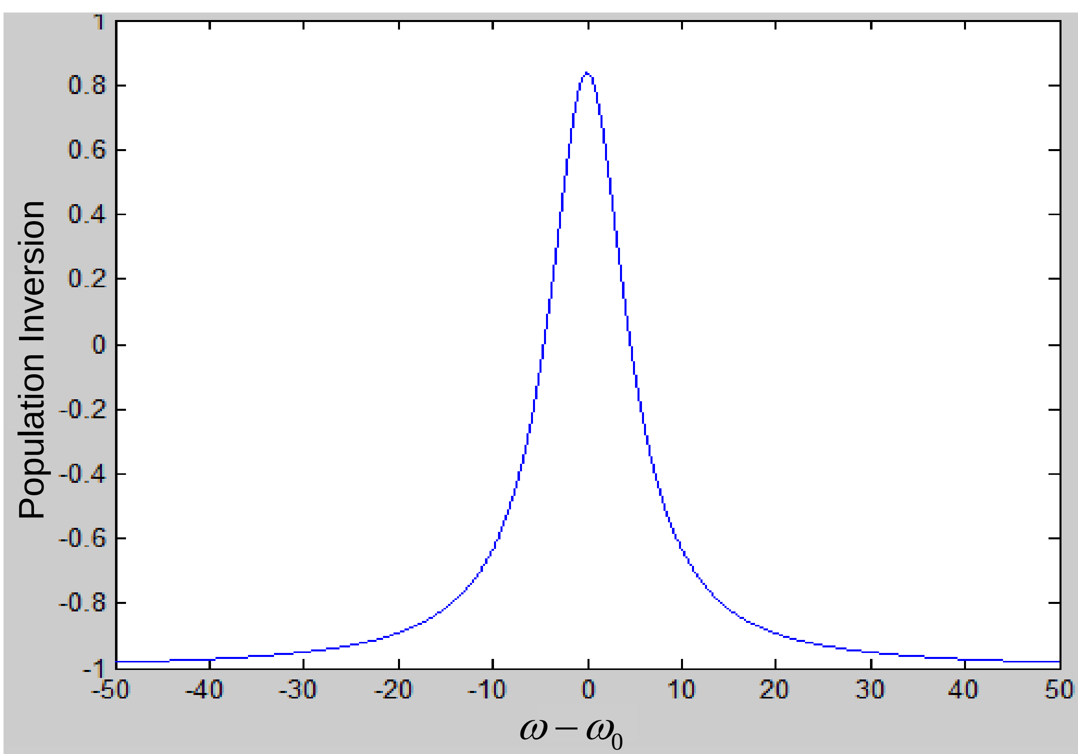

Figure 4. Average population inversion with $\omega - \omega_0$

In the derivation of the equation, all the electrons were considered initially to be at the lower or non-excited state at $t=0$. Therefore the initial population inversion is -1. The population inversion varies sinusoidal with time. The frequency of this sinusoidal variation is a maximum at the threshold frequency ω_0 of the atom. As the electric field frequency move away from the threshold frequency, the magnitude of the population inversion decreases rapidly. The Figure 3 represents the electron population of the lower energy state. The average population inversion with $\omega-\omega_0$ is shown in Figure 4. The average population inversion, a maximum when the field frequency is equal to the resonance frequency of the medium, decreases considerably for a slight deviation of the field frequency from resonance frequency of the medium.

3. THE GAIN AND PUMP PARAMETER

Substituting the imaginary part of $\rho(t)_{avg}$ obtain in the equation 40 in the equation of motion of the laser field given in equation 25 can be written as

$$\dot{\varepsilon}(t) = \left(\frac{c^2\mu_0\omega\gamma N(\mathbf{r})|D_{12}|^2}{4\hbar((\omega-\omega_0)^2+\gamma^2/4)} - \frac{1}{2}c^2\mu_0\sigma\right)\varepsilon(t) - \frac{c^2\mu_0\omega\gamma^3 N(\mathbf{r})|D_{12}|^4}{4\hbar^3\left((\omega-\omega_0)^2+\gamma^2/4\right)^3}|\varepsilon(t)|^2\varepsilon(t) \qquad (41)$$

$$\dot{\varepsilon}(t) = \left(A-C\right)\varepsilon(t) - B|\varepsilon(t)|^2\varepsilon(t) = A\left(b-\frac{B}{A}|\varepsilon(t)|^2\right)\varepsilon(t) \qquad (42)$$

where A, C, B are the coefficients of gain, loss and non-linear terms respectively and b is the pump parameter:

$$A=\frac{c^2\mu_0\omega\gamma N(\mathbf{r})|D_{12}|^2}{4\hbar((\omega-\omega_0)^2+\gamma^2/4)}; \quad B=\frac{c^2\mu_0\omega\gamma^3 N(\mathbf{r})|D_{12}|^4}{4\hbar^3\left((\omega-\omega_0)^2+\gamma^2/4\right)^3}; \quad C=\frac{1}{2}c^2\mu_0\sigma; \quad b=\frac{A-C}{A}$$

The pump parameter b is positive above threshold and negative below the threshold. For a steady state solution putting $d\varepsilon(t)/dt=0$,

$$\begin{aligned} |\varepsilon(t)|^2 &= Ab/B \quad b\geq 0; \\ \varepsilon(t) &= 0 \qquad\quad b\leq 0 \end{aligned} \qquad (41)$$

The behaviour of gain co-efficient A with the frequency of the cavity electric field is shown in Figure 5. The resonance frequency of the two laser levels was taken as $1000\,\text{kHz}$. Similar to the population inversion, the gain co-efficient also has a sharp peak at the resonance frequency of the atoms. It can be seen that when the field frequency move away from this resonance frequency the gain co-efficient rapidly reduce to zero, which stops the laser action.

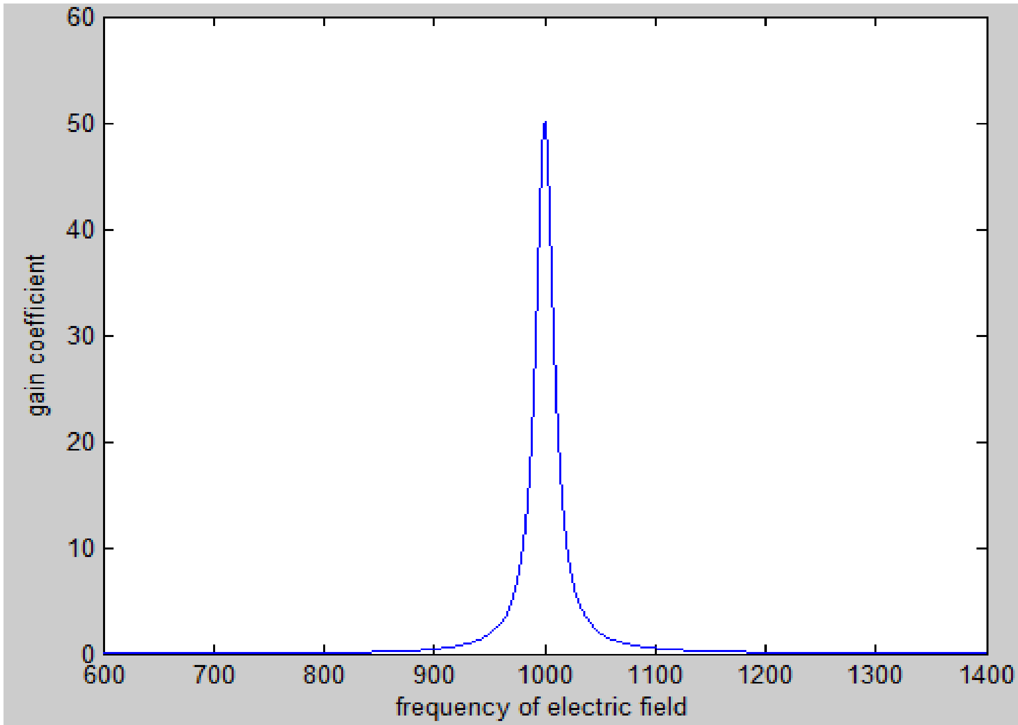

Figure 5. The gain co-efficient with the frequency of the cavity electric field ($\boldsymbol{\omega_0 = 1000\ kHz}$).

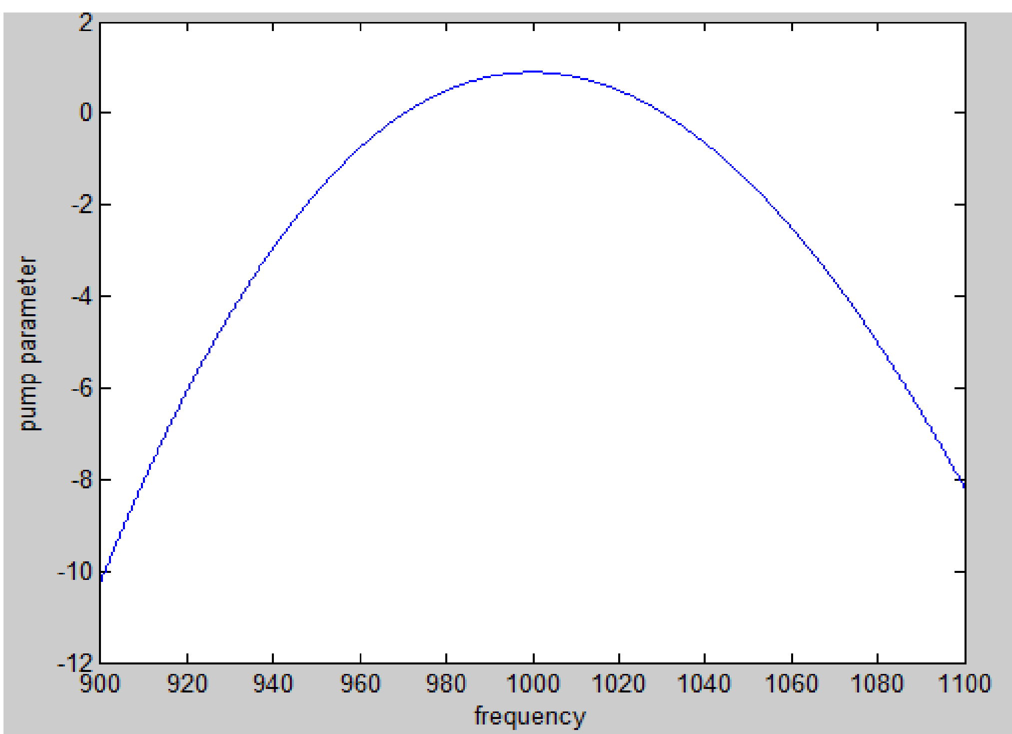

Figure 6. The pump parameter with the frequency of the cavity electric field ($\boldsymbol{\omega_0 = 1000\ kHz}$).

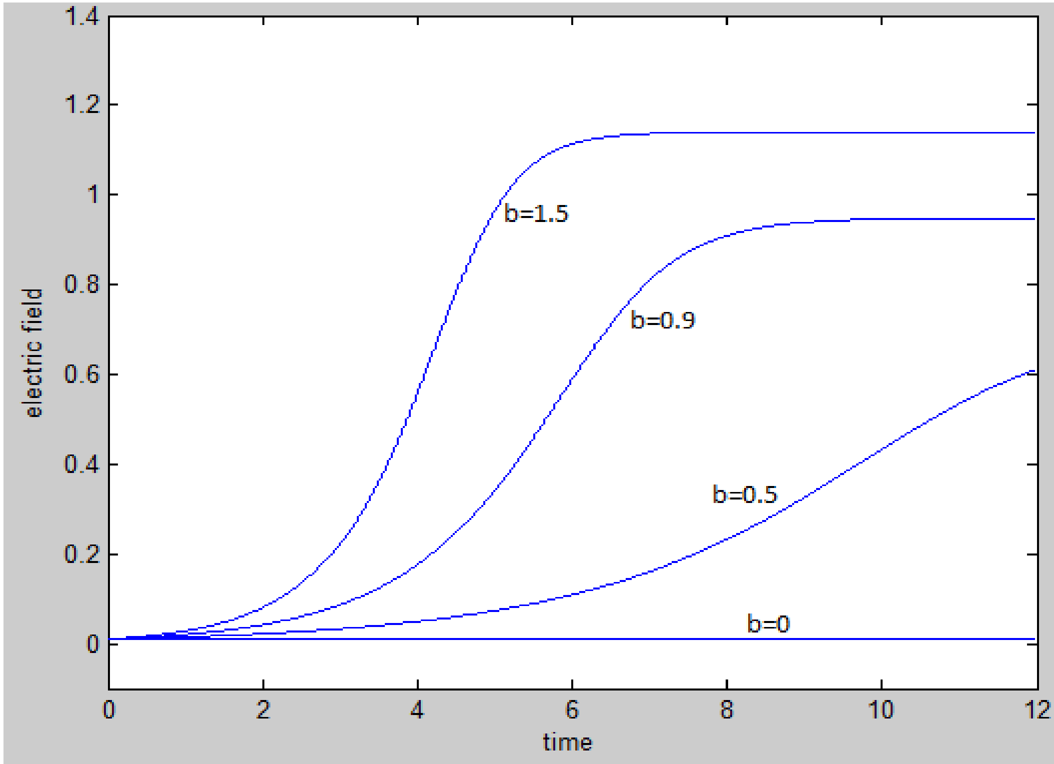

Figure 7. The electric field with time for pump parameters $b = 0,\ 0.5,\ 0.9,\ 1.5$

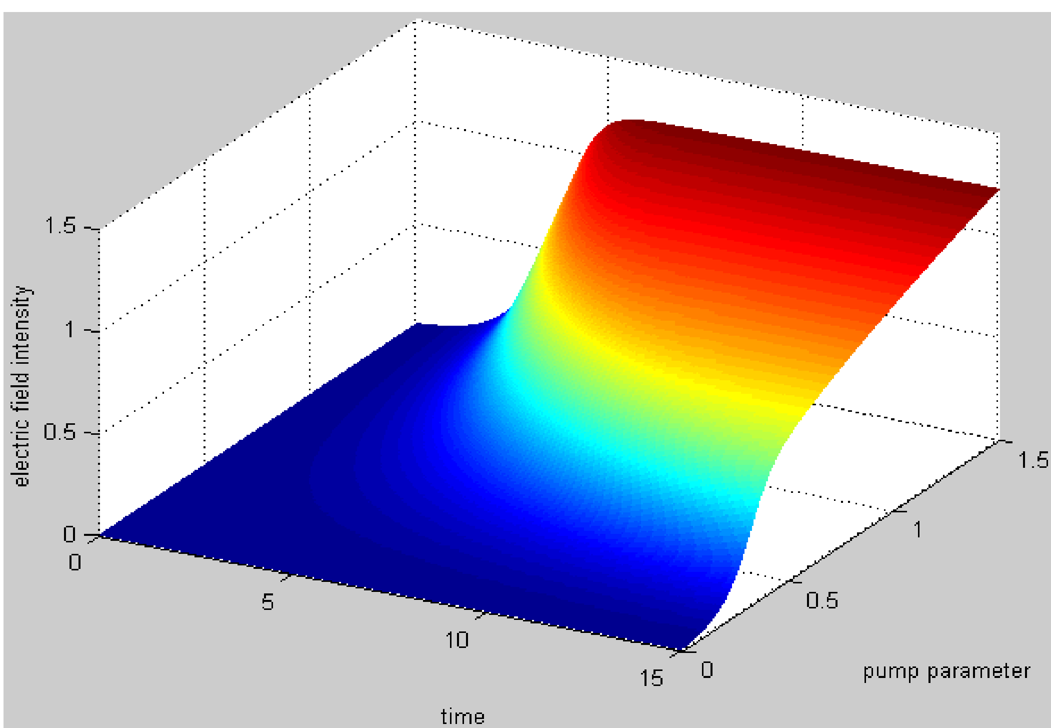

Figure 8. The electric field with time for pump parameters $b = 0 - 1.5$ and time

The Figure 6 shows the pump parameter with the cavity field frequency. The pump parameter is positive only around the resonance frequency of the medium and at the resonance frequency it has the maximum value. When the pump parameter is negative, the loss is greater than the gain. Figure 7 shows the electric field intensity inside the laser cavity with time, for different pump parameters $b = 0$, 0.5, 0.9 and 1.5. The electric field intensity inside the laser cavity with time, for the pump parameters in the rage $b = 0$ to 1.5 is shown in figure 8. When the pump parameter is equal to zero, in other words the gain is equal to the loss, the electric field remains zero over time. It can be observed that when *b* is increased the saturation electric field is also increased. For larger values of the pump parameter, smaller time is needed to reach the saturation of the laser field. The saturated intensity of laser field with the pump parameter is shown in Figure 9. When the pump parameter is less than or equal to zero the saturated intensity is zero. But when pump parameter is positive, the saturated intensity is proportional to pump parameter. The slope is equal to A/B where A is the gain co-efficient and B is the nonlinear term.

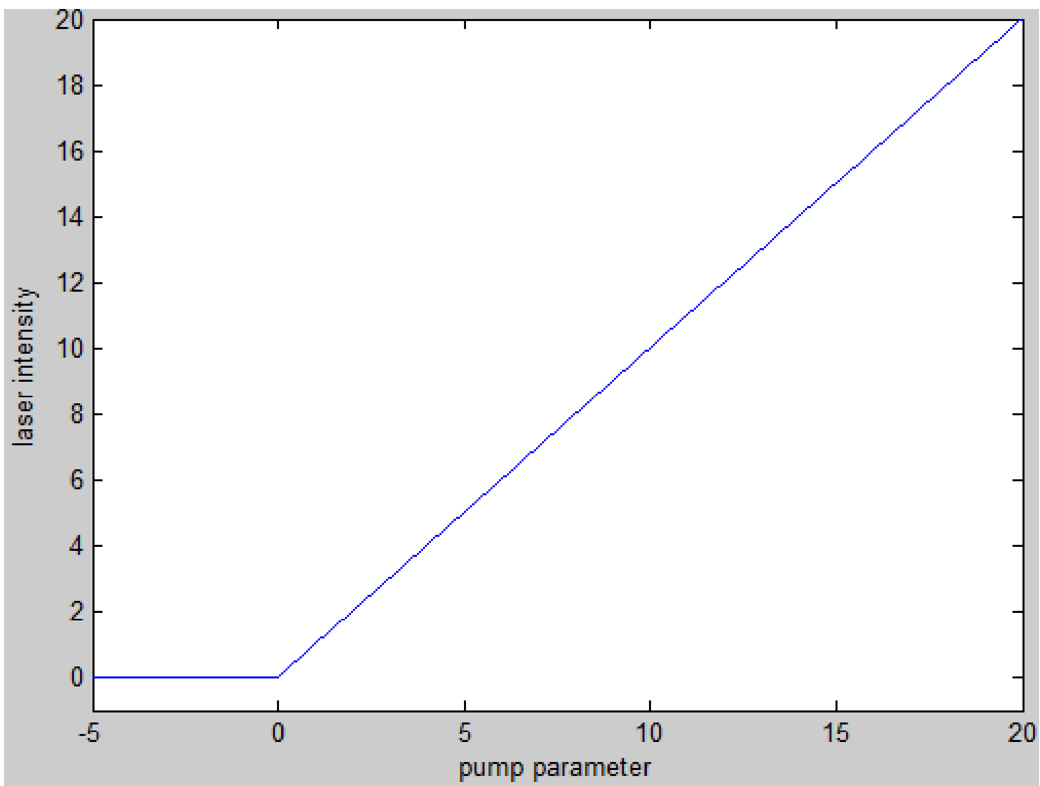

Figure 9. The saturated laser intensity at steady state with pump parameter *b*

4. CONCLUSION

In semi classical model, for the population inversion, peak intensity is obtained when the field frequency is equal to the resonance frequency of the atoms in the laser cavity. Above the threshold when the gain exceeds the loss or in other words pump parameter is positive, the electric filed inside the optical cavity grows exponentially, until the saturation term begins to dominate and inhibit the further growth. The initial growth depends on the presence of a nonzero field at the beginning, and in absence of an electric field at the beginning no matter how high the population inversion is, the electric field remain zero for all the time.

References

[1] A. Einstein, *Physikalische Gesellschaft Zürich* 18 (1917) 121-128.

[2] P. Gordon, H. J. Zeiger, and C. H. Townes, *Phys. Rev*. 95 (1954) 282.

[3] R. J. Glauber, *Phys. Rev*. 131(6) (1963) 2766-2788.

[4] W. E. Lamb, *Phy. Rev*. 134 (1965) 1429.

[5] A.G. fox, T. Li, *Bell Sys. Tech. Jour.* 40(2) (1961) 453-488.

(Received 09 June 2015; accepted 23 June 2015)