Available online at www.worldscientificnews.com

World Scientific News

WSN 12 (2015) 12-26 EISSN 2392-2192

F(T) Dark Energy Model and SNe Data

S. Davood Sadatian[a], Amin Anvari[b]

Department of Physics, Faculty of Basic Sciences, University of Neyshabur,
P. O. Box 9318713331, Neyshabur, Iran

[a,b]E-mail address: sd-sadatian@um.ac.ir , a-anvari@neyshabur.ac.ir

ABSTRACT

We study a general F(T) gravity theory for considering cosmological Dark energy scenario. In this regards, we use Torsion scalar (T) in gravitation sector of lagrangian to obtain main cosmological parameters such as: equation of state, declaration parameter, acceleration and Hubble parameter. In following with suitable choice of F(T) function, we compare the model behaviors with observational data of universe evolution.

Keywords**:** F(T) gravity; Dark energy; Hubble Parameter; Deceleration Parameter

PACS: 95.36.+x, 98.80.-k, 04.50.kd

1. INTRODUCTION

In present era, many cosmological observational data such as CMB(cosmic microwave background radiation), BAO(baryon acoustic oscillations), SN Ia(supernovae Ia) prove our universe has an accelerated expansion [1-5]. For explanation this subject, many authors suggested some different models, an interesting solution is called Dark energy (DE).

Important subject in this area is construction of Dark energy Models. One of these models is cosmological constant Λ with EoS(equation of state) $\omega = \frac{p}{\rho} = -1$, it is simplest model. However, this model has two important problems i.e. the cosmological constant problem and the coincidence problem. Various dynamical Dark energy models - quintessence and phantom - have been prepared to solve these problems [6]. Also holographic Dark energy is new suggestion for explanation dynamical Dark energy models [7].

In other view point, One can modifies Einstein equation to crate other cosmological model that is called F(R) gravity. In this framework, Ricci scalar or a function of Ricci scalar used in Einstein-Hilbert action [8]. However, scalar field can couple non-minimally or minimally with Ricci scalar to explain some properties of DE [9-12]. But some conditions in F(R) theories cause a singularity in large Ricci scalar (R). Although this problem needs more study, it seems for avoid this singularity,we should use other alternative model. However, this is probably the big challenge for F(R) theories [13-15]. Based on the above description, we need a suitable model.

An interesting alternative proposal is F(T) gravity. This model use torsion scalar (T) in density lagrangian instead of Ricci scalar [16-18]. F(T) gravity model can use minimal or non-minimal coupled torsion scalar with gravity [19-22]. In following, we use a cosmological F(T) gravity model to determine some major parameters in cosmology. Finally, we compare our results with observational data. In addition, we point out one significant category of dark energy models that had greatly developed the knowledge about dark energy. To be specific, dark energy could either be quintessence-like, phantom-like, or the so-called "quintom"-like.

This quintom type of dark energy model was proposed in [23], and then extensively developed in a number of works, such as [24]. The models of quintom type suggest that the equation of state parameter of dark energy can cross the cosmological constant boundary w = – 1. It is interesting to note that this quintom scenario has exactly appeared in this paper.

Therefore, structure of this paper is : In section 2, we briefly introduce teleparallel Dark energy model F(T) and then we derive general equations to study behavior of the model(according F(T) form). In section 3, we consider some reasonable F(T) in proposal model to obtain some cosmological aspects, then parameters of model compare with observational data. In section 4, we summarize our results.

2. THE F(T) GRAVITY MODEL

Here, we first review the teleparallel gravity formalism according[25]. The torsion scalar define as

$$T = S_{\rho}{}^{\mu\nu} T^{\rho}{}_{\mu\nu}, \tag{1}$$

$$S_{\rho}{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_{\rho} + \delta^{\mu}_{\rho} T^{\theta\nu}{}_{\theta} + \delta^{\nu}_{\rho} T^{\theta\mu}{}_{\theta}\right), \tag{2}$$

$$T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\nu\mu} - \Gamma^{\lambda}{}_{\mu\nu} = h_i^{\lambda}\left(\partial_{\mu} h^i_{\nu} - \partial_{\nu} h^i_{\mu}\right), \tag{3}$$

and the contorsion tensor $K_{\rho}^{\mu\nu}$ is

$$K^{\mu\nu}{}_{\rho} = -\frac{1}{2}(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}), \tag{4}$$

where h_{μ}^{i} are the components of the non-trivial tetrad field h_i in the coordinate basis. The metric tensor $g_{\mu\nu}$ take

$$g_{\mu\nu} = \eta_{ij} h_{\mu}^{i} h_{\nu}^{j}, \tag{5}$$

where $\eta_{ij} = diag(1,-1,-1,-1)$ is the Minkowski metric for the tangent space. For infinite different tetrad fields h_{μ}^{i} the following condition exist :

$$h_{\mu}^{i} h_{j}^{\mu} = \delta_{j}^{i},$$

$$h_{\mu}^{i} h_{i}^{\nu} = \delta_{\mu}^{\nu}. \tag{6}$$

It should stress that $(i, j, ... = 0,1,2,3)$ used to denote the tangent space indices and $(\mu, \nu, ... = 0,1,2,3)$ to denote the spacetime indices.

The action of teleparallel gravity is given

$$S = \int d^4x\, e[\frac{F(T)}{2k^2} + L_m] \tag{7}$$

where $e = det(e_{\mu}^{i}) = \sqrt{-g}$ and $k^2 = 8\pi G = \frac{1}{M_{pl}^2}$, while G is a gravitational constant and M_{pl} is a reduced Planck mass. With using Eq.(7), one can obtain the following field equation

$$[e^{-1}\partial_{\mu}(eS_i{}^{\mu\nu}) - h_i^{\lambda} T^{\rho}{}_{\mu\lambda} S_{\rho}{}^{\nu\mu}] F_T + S_i{}^{\mu\nu} \partial_{\mu}(T) F_{TT} + \frac{1}{4} h_i^{\nu} F = \frac{1}{2} k^2 h_i^{\rho} T_{\rho}^{\nu}, \tag{8}$$

here, $F_T = \frac{dF}{dT}$, $F_{TT} = \frac{d^2F}{dT^2}$, $S_i{}^{\mu\nu} = h_i^{\rho} S_{\rho}{}^{\mu\nu}$ with antisymmetric property and $T_{\mu\nu}$ is the energy-momentum tensor

$$T_{\rho}^{\nu} = diag(\rho, -p, -p, -p), \tag{9}$$

where ρ, p are density and pressure of context in universe respectively.

The FRW metric has form as

$$ds^2 = dt^2 - a(t)^2 dx^2 - a(t)^2 dy^2 - a(t)^2 dz^2, \tag{10}$$

where a(t) is the scale factor.

By substituting Eqs.(2) and (3) in Eq.(1) and using Eq.(10), we can obtain

$$T = -6H^2, \tag{11}$$

where H is the Hubble parameter. However, by setting F(T)=T, the Friedmann equations obtain as [26]

$$H^2 = \frac{1}{3}k^2\rho$$

$$\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} = k^2 p$$

Following, we take a general form of Friedmann equations and using F(T) and its differentials in these relations(these equations consider in specific form of F(T) in next section.), therefore, by using Eq.(11), we have [25]

$$F - 12H^2 F_T = 2k^2\rho, \tag{12}$$

$$(8H^2 + 4\frac{\ddot{a}}{a})F_T - 48H(\frac{\ddot{a}}{a} - H^2)F_{TT} - f = 2k^2 p. \tag{13}$$

Now the conservation law with an energy density ρ and a pressure p can write

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{14}$$

where dark energy density ρ and pressure p given by following relations

$$\rho_{de} = \frac{1}{2k^2}[-12H(t)^2(1 - F_T) + T - F] \tag{15}$$

$$p_{de} = \frac{1}{2k^2}[4(2H(t)^2 + \frac{\ddot{a}}{a})(1 - F_T) + 48H(t)^2(\frac{\ddot{a}}{a} - H(t)^2)F_{TT} - T + F] \tag{16}$$

Therefore, the equation of state parameter $\omega_{de} = \frac{P}{\rho}$ for Dark energy obtain

$$\omega_{de} = \frac{E(1-F_T)+YF_{TT}-T+F}{Z(1-F_T)+T-F} \tag{17}$$

where

$$E = 4(2H(t)^2 + \frac{\ddot{a}}{a}), \quad Y = 48H(t)^2(\frac{\ddot{a}}{a} - H(t)^2), \quad Z = -12H(t)^2$$

By using definition of deceleration parameter q, such as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}, \quad q = -(1+\frac{\dot{H}}{H^2}), \quad q = \frac{1}{2}(1+3\omega).$$

The deceleration parameter obtain in following relation

$$q = \frac{1}{2} + \frac{3}{2}\frac{E(1-F_T)+YF_{TT}-T+F}{Z(1-F_T)+T-F} \tag{18}$$

Finally, one can calculate Hubble parameter and scale factor or take an ansatz for these relations in numerical consideration. Now, we have essential cosmological equations of the model, in following with using above equations, we can study some behaviors of F(T) model and compare our results with observational data.

3. CONSIDERING PARAMETERS OF MODEL ACCORDING UNIVERSE EVOLUTION

In this section, first we should determine explicit form of F(T) function. Therefore, we choose three cases of F(T) and consider dynamical relations. However, these considerations are valid if the behaviors of model according to cosmic evolution.

Case I : $F(T) = \alpha(-T)^n$ [26]

Phenomenological reason for choice this case is a de Sitter fate for the universe; here, we take $n = \frac{1}{2}$. By using equations in section 2, in Figures (1) and (2) we show dynamical aspects of parameters. As we shown, the equation of state parameter crossing phantom divide line that satisfy by data. We know that the universe is expanding and deceleration should be near to zero, this is appear in Fig (2).

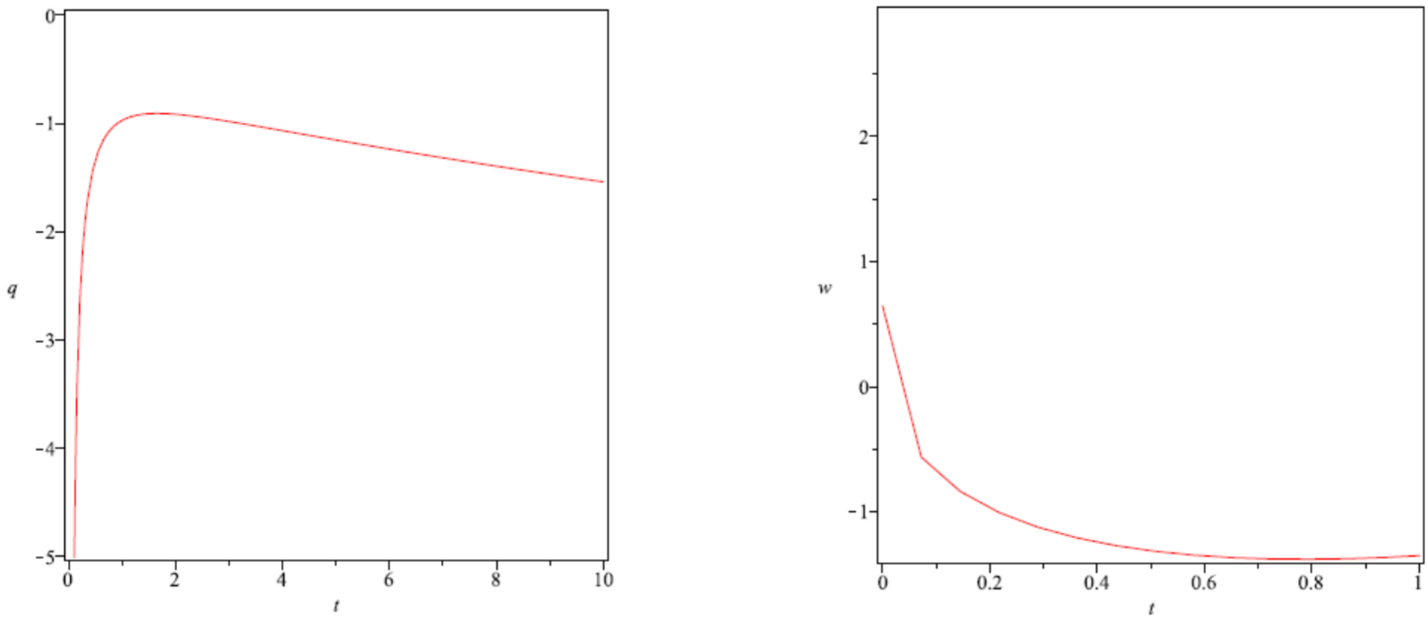

Figure 1. (a) Evolution of Hubble parameter H(t) and scale factor a(t).

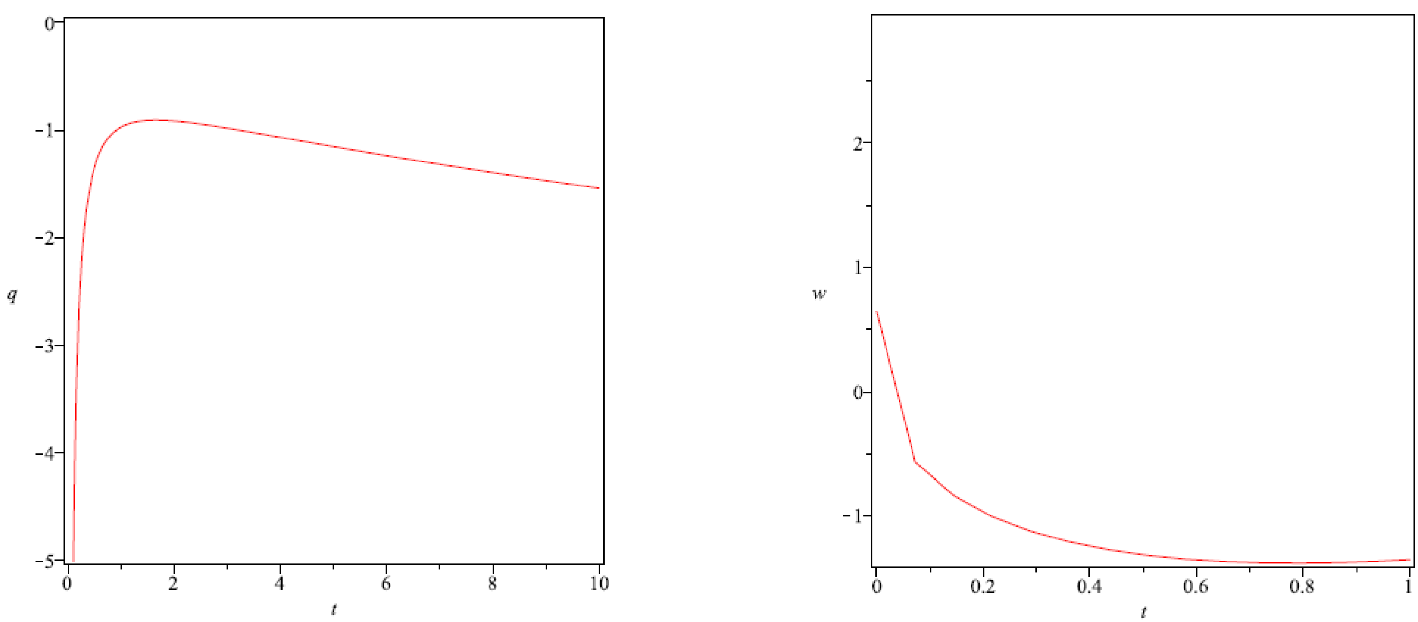

Figure 2. (b) Deceleration parameter q(t) and equation of state $\omega(t)$ for $n = \frac{1}{2}$ and $\alpha = 1$

Case II: $F(T) = \alpha(-T)^n tanh(\frac{T_0}{T})$ [26]

Physical caution of this case is possible expression that leading to an accelerated expansion phase. It is particularly interesting to look at models which are able to give an effective EoS with crossing the phantom divide. Following, we refer to a tanh model, one should set $n > 3/2$ for having a dark torsion fluid with a positive energy density. Also interpreting gravitational interactions in terms of the torsion rather than the scalar curvature cause to the equivalent teleparallel formulation of General relativity. especially, for some suitable choices of the F(T), it is possible to get an accelerating expansion in a matter only universe with the effective torsion fluid that playing the role of dark energy in context of model. In Figures (3) and (4), we have convincingly shown that the model is in excellent agreement with cosmological evolution.

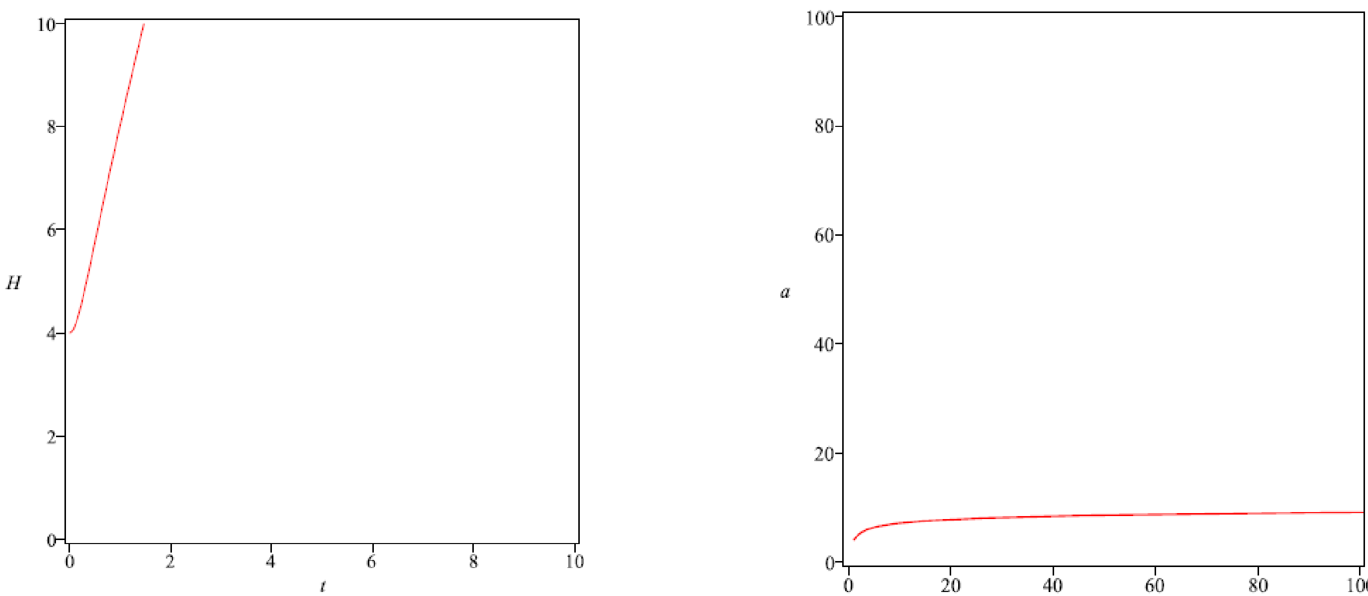

Figure 3. (c) Evolution of Hubble parameter H(t) and scale factor a(t)

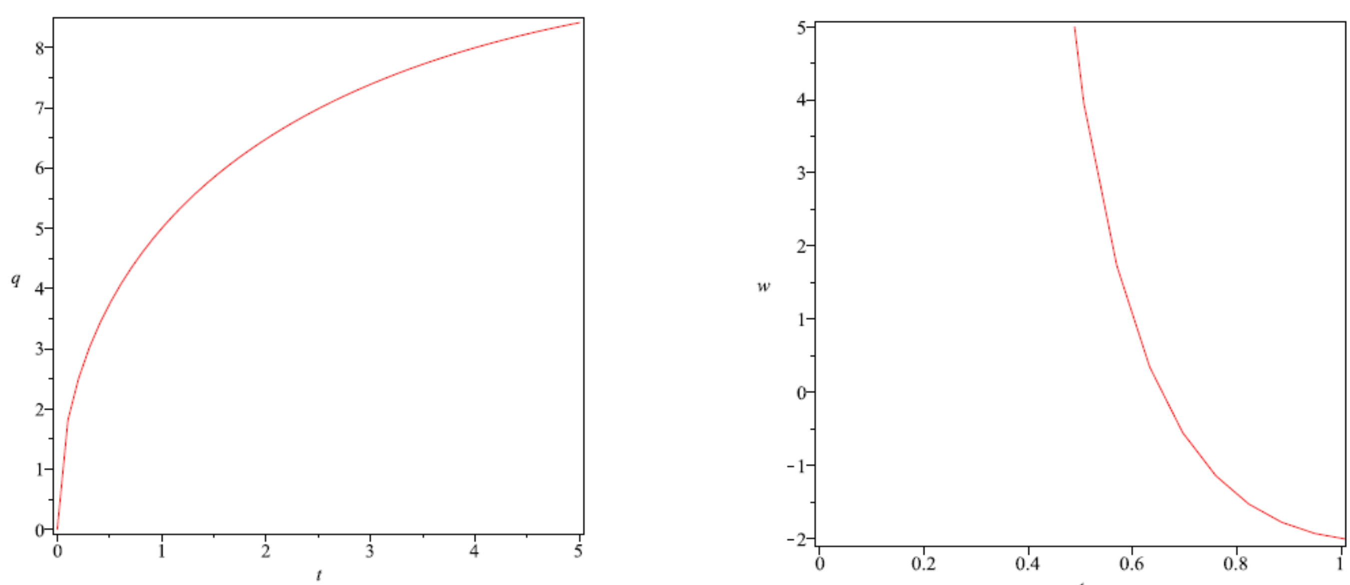

Figure 4. (d) Deceleration parameter q(t) and equation of state $\omega(t)$ for $n = 2$ and $\alpha = T_0 = 1$

Case III: $F(T) = T + \alpha T_0 (1 - e^{\frac{\beta T^2}{(T_0)^2}})$ [27]

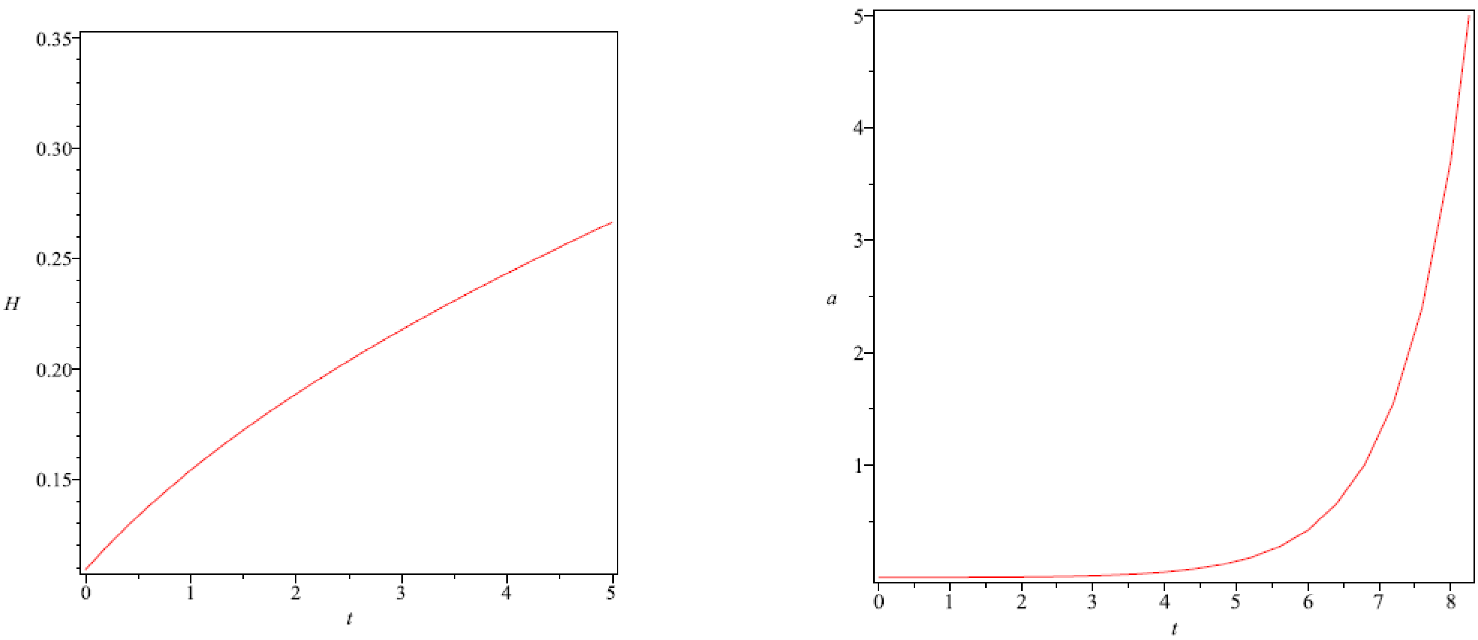

Figure 5. (e) Evolution of Hubble parameter H(t) and scale factor a(t).

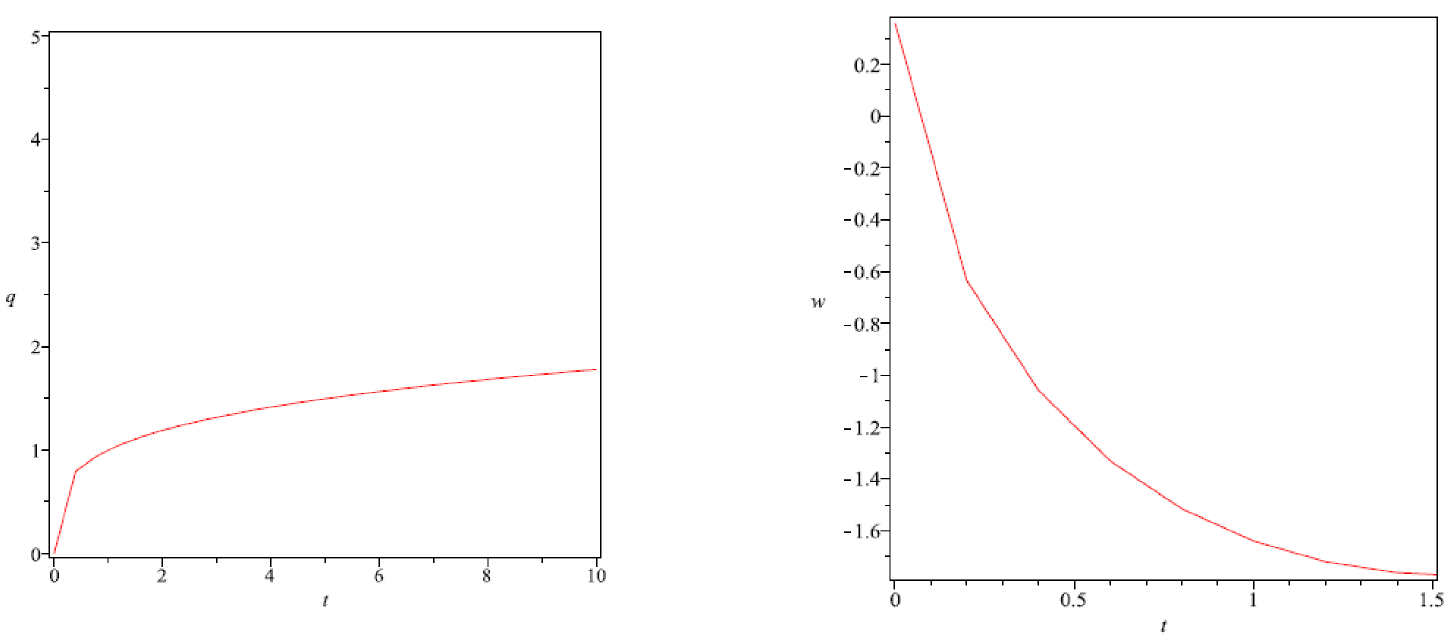

Figure 6. (f) Deceleration parameter q(t) and equation of state $\omega(t)$ $\alpha = T_0 = 1$ and $\beta = -1$

About this F(T) function, we should emphasize this choice of F(T) can be reduced back to TEGR (teleparallel equivalence of general relativity),this means for small T, corresponding to the current universe. Obviously, when T is very small compared to T_0 and $\beta < 0$ the model get $F(T) \approx T$ (this approximation considered in section 2). for an exponential gravity model, we can obtain the stable solution for the open (closed) Einstein universe (for more details see Appendix). these discussions shown in Figures (5) and (6). Also, we can see equation of state parameter crossing phantom divide line by this choice of F(T) function.

3. 1. Comparison with Observational Data

As we shown in previous section for some choices of F(T) function, we can consider behavior of cosmological parameters. In following, we compare our result with recent cosmological observation data. Hence, with using some data such as (SNe Ia, CMB, LSS, BAO, WMAP, SDSS), we conclude the expansion of the universe is accelerating at the present epoch [28,29]. In following, we focus in SNe Ia data. Also, these data show equation of state ω crossing into the phantom divide in the universe evolution.

The type Ia supernova is a suitable tool to consider the expanding rate of the universe. In brief detail, type SNe Ia is a supernova which just reaches the Chandrasekhar bound and then explodes. Here, we show some results of one of the best recent sample, Union [30]. SNe Ia data show that: 1. the universe is almost spatially flat, 2. the present universe is dominated by dark energy, whose partition is approximately 70 percent, and the partition of dust is 30 percent.

Also, we know that the EoS of dark energy is not exactly equal to -1. The fitting results by different samples of SNe Ia are shown in fig 7. We see that although a cosmological constant is permitted, the dark energy whose $EoS < -1$ is requested by SNe Ia. We probably seem that phantom is a candidate for the dark energy whose EoS less than -1.

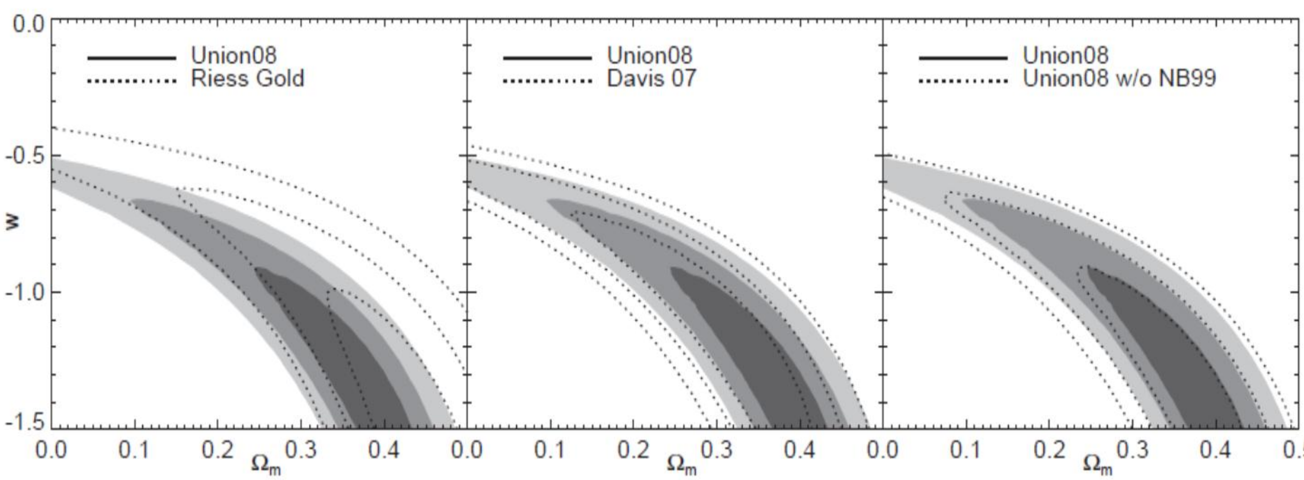

Figure 7. (g) The EoS of dark energy fitted by SNe Ia in a spatially flat universe. The results of the Union set are shown as filled contours (The numerical data of the this sample are also available in [30])

The perturbation of the dark energy will increase if its EoS is not exactly -1 in the evolution history of the universe. Hence, for fitting a dynamical dark energy model with observation, the perturbation of the dark matter should be studied in principle. With more and accurate data, the probability of crossing -1 (phantom divide) seems a more specific. This crossing manner is a challenge for theoretical physics. In this regards, the EoS of dark energy can not cross the phantom divide if: 1. a dark energy component with an arbitrary scalar field Lagrangian or other extra function, which has a general dependence on the field itself, 2. general relativity holds and 3. the spatially flat Friedmann universe [31]. However, as we shown in this article, equation of state can across the phantom divide line in F(T) gravity (by a suitable choice of F(T)) and acceleration expansion occur.

As we know, some other fate for universe such a Rip singularity and bouncing model can realize. For example, if our universe crosses $\omega_{DE} = -1$, then its future can be really Dark. This means, universe may enter in a future singularity(for example Rip Singularity) or an interesting solution of the singularity problem in cosmology as Bouncing model. For the universe coming into the Big Bang era after the bouncing, the equation of state parameter should crossing from $\omega < -1$ to $\omega > -1$. The universe has its smallest expanse (smallest scale factor a(t)) and largest energy density at this situation [32,33]. following we check our model and illustrated the evolution in terms of the redshift instead of the cosmic time and calculated the maximum likelihood between observational data (SNe Ia) and parameters of model.

3. 2. SNe Data and F(T) gravity

The parameters of the cosmological models can be fitted from a exact comparison of their results with observational data. Here, we consider the data coming from SNe observations with parameters of F(T) model(the evolution of the Hubble parameter). The modulus μ vs redshift $z = a_0/a - 1$ equation corresponding to type Ia supernovae from the Supernova Cosmology Project[30] is, as well known,

$$\mu(z) = \mu_0 + 5\lg D_L(z).$$

The equation for the luminosity distance $D_L(z)$ as a function of the redshift in the FRW cosmology give

$$D_L^{FC} = \frac{c}{H_0}(1+z)\int_0^z h^{-1}(z)dz, \tag{19}$$

where $h(z) = \left[\Omega_{m0}(1+z)^3 + \Omega_{D0}F(z)\right]^{1/2}$.

Here, Ω_{m0} is the total content of matter density, Ω_{D0} the content of DE energy density, and H_0 the Hubble parameter in current era. The constant value μ_0 depends on the specific Hubble parameter:

$$\mu_0 = 42.384 - 5\log h, \quad h = H_0/100km/s/Mpc$$

The function $F(z) = \rho_D(z)/\rho_{D0}$ can be obtained from the continuity equation

$$\dot{\rho}_D - 3\frac{\dot{a}}{a} g(\rho_D) = 0, \tag{20}$$

which can be reproduct in form of

$$\int_{\rho_{D0}}^{\rho_D(z)} \frac{dy}{g(y)} = -3\ln(1+z). \tag{21}$$

For simplicity, we ignore the contribution of radiation. For F(T) cosmology, Eq.(19) can be rewritten

$$D_L^{BC} = \frac{c}{H_0}(1+z)\int_0^z h^{-1}(z)[1+\delta h^2(z)]^{-1/2}(1+\delta)^{1/2} dz$$

where for convenience the parameter $\delta = \rho_0/2$ has been introduced. For the analysis of the SNe data we need to determine the parameter χ^2, which is defined with

$$\chi_{SN}^2 = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i)]^2}{\sigma_i^2}, \tag{22}$$

where σ_i is 1σ error. The parameter μ_0 is independent of the data points and, hence, we have to perform a uniform marginalization over μ_0. Minimization with respect to μ_0 can be done by expanding the χ_{SN}^2 with respect to μ_0,

$$\chi_{SN}^2 = A - 2\mu_0 B + \mu_0^2 C, \tag{23}$$

where

$$A = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0)]^2}{\sigma_i^2},$$

$$B = \sum_i \frac{\mu_{obs}(z_i) - \mu_{th}(z_i)}{\sigma_i^2}, \quad C = \sum_i \frac{1}{\sigma_i^2}.$$

This term has a minimum for $\mu_0 = B/C$ at

$$\bar{\chi}^2_{SN} = A - B^2/C.$$

In following, we should minimize $\bar{\chi}^2_{SN}$ instead of χ^2_{SN}. Now by data in ref[30] and table 1, we obtain the 43.27% confidence level by $\Delta\chi^2 = \chi^2 - \chi^2_{min} < 1.23$ for the one-parameter or 1.72 for the two-parameter model. Similarly, the 82.09% confidence level is obtained with $\Delta\chi^2 = \chi^2 - \chi^2_{min} < 2.11$ for one-parameter and 4.08 for the two-parameter models.

Table 1. Some amounts of Hubble parameter versus redshift data.

z	$H_{obs}(z)$ km s^{-1} Mpc^{-1}	σ_H km s^{-1} Mpc^{-1}
0.170	83	8
1.530	140	14
1.750	202	40

3. 3. F(T) gravity and other Dark energy Models

Until now, we consider a F(T) model that explain dark energy in content of the universe, but according Refs [32,33 and references therein], there are many different models in this subject have been studied. Therefore, in this subsection for a comparison review, we briefly reconsider them.

As we know many popular dark energy models, such as the ΛCDM model, Little Rip and Pseudo-Rip scenarios, the phantom and quintessence cosmologies with the four types (I, II, III and IV) of the finite-time future singularities and non-singular universes filled with dark energy have been studied. Results show the ΛCDM model and different cosmological observations to determine the bounds on the late-time acceleration of the universe. In addition, authors have considered a fluid description of the universe in which the dark fluid has a general form of the EoS.

They have shown that all the dark energy models can be constructed by different fluids. In other hand, they have considered the equivalence of other dark energy models, such as: single and multiple scalar field theories. In this regards, tachyon scalar theory and holographic dark energy, in which the quintessence/phantom model by the current acceleration can be provided, and compared those equivalence to the corresponding fluid methods. Furthermore, other equivalent class of dark energy models, in which dark energy has its geometrical origins, this article is one of them (F(T) models).

In all of the above models, the difference is just the forms of the energy density ρ_{DE} and pressure P_{DE} of dark energy. therefore, the Hubble parameter H relate to the concrete cosmology, such as, the ΛCDM, quintessence and phantom models, can be reproduced with using gravitational field equations or by applying ρ_{DE} and P_{DE}, the EoS $w_{\mathrm{DE}} \equiv \mathrm{DE}/\rho_{\mathrm{DE}}$ in the fluid form can be presented.

However, we emphasize the role of cosmography in this discussion. As shown, it is a basic tool because it permits, to separate among models without initial assumptions but just import constraints coming from cosmological evolution.

4. SUMMARY

The teleparallel dark energy scenario [19-25] is based on the teleparallel analogous of general relativity (TEGR). We have considered a F(T) teleparallel dark energy model where torsion parameter played the role of scalar field (Dark Energy).

An interesting feature of the our model is that it realizes the phantom divide line crossing for equation of state, By considering three types of F(T) functions, we found that ω can cross the -1 line as shown in Figures 1-6. Furthermore, we also found evolution of other cosmological parameters such as Hubble variable and declaration parameter.

We shown our results coincide with observational data and calculated the maximum likelihood. In summary, our model has the property of wide rang of parameters which one use fine-tuning mechanism for fitting model by data.

References

[1] D N Spergel et al., *Astrophys. J. Suppl.* **148** 175 (2003).

[2] S Perlmutter, et al., *Astrophys. J.* **517** 565 (1999).

[3] A G Riess et al., *Astron. J.* **116** 1009 (1998).

[4] D J Eisenstein et al., *Astrophys. J.* **633** 560 (2005).

[5] E Komatsu et al., *Astrophys. J. Suppl.* **192** 18 (2011).

[6] J Yoo and Y Watanabe, *Int. J. Mod. Phys. D.* **21** 1230002 (2012).

[7] A Banijamali, M.R. Setare and B. Fazlpour, *Int. J. Theor. Phys.* **50** 3275 (2011).

[8] Sh Nojiri and S D Odintsov, *Phys. Rept.* **505** 59 (2011).

[9] S Capozziello and M D Laurentis, *Phys Rept.* **509** 167 (2011).

[10] T P Sotiriou and V Faraoni, *Rev. Mod. Phys.* **82** 451 (2010).

[11] A Guarnizo, L Castaneda and J M Tejeiro, *Gen. Rel. Grav.* **42** 2713 (2010).

[12] T Prokopec and J Weenink, *JCAP* **12** (2012).

[13] V Faraoni, *Presented at SIGRAV2008, 18th Congress of the Italian Society of General Relativity and Gravitation, Cosenza, Italy September 22-25* (2008).

[14] A V Frolov, *Phys. Rev. Lett.* **101** 061103 (2008).

[15] T Kobayashi and K Maeda, *Phys. Rev. D.* **78** 064019 (2008).

[16] E E Flanagan and E Rosenthal, *Phys. Rev. D.* **75** 124016 (2007).

[17] G Bengochea and R Ferraro, *Phys. Rev. D.* **79** 124019 (2009).

[18] K K Yerzhanov et. al., [arXiv:1006.3879].

[19] T Clifton et. al., *Physics Reports.* **513** 1 (2012).

[20] A Banijamali and B Fazlpour, *Astrophysics and Space Science.* **342** 229 (2012).

[21] H Wei, X P Ma and H Y Qi, *Phys. Lett. B.* **703** 74 (2011).

[22] Ch Q Geng et. al., *Phys. Lett. B.* **704** 384 (2011).

[23] B Feng, X Wang and X Zhang, *Phys. Lett. B.* **607** 35 (2005).

[24] Y-F Cai, H Li, Y-S Piao and X Zhang, *Phys. Lett. B.* **646** 141 (2007);
Y-F Cai and J. Wang, *Class. Quant. Grav.* **25** 165014 (2008).

[25] M Sharif and S Rani, *Mod. Phys. Lett. A.* **26** 1657 (2011).

[26] E V Linder, *Phys. Rev. D.* **81** 127301 (2010).

[27] J T Li, Ch Ch Lee and Ch Q Geng, *Eur. Phys. J. C.* **73** 2315 (2013).

[28] J Frieman, M Turner and D Huterer, *Ann. Rev. Astron. Astrophys.* **46** 385 (2008).

[29] D H Weinberg et al., *Phys Report.* **530** 87 (2013).

[30] http://supernova.lbl.gov/Union

[31] H Zhang, *Dark Energy: Theories, Developments and Implications , Nova Science Publisher* 49 (2010).

[32] K Bamba et al., *Astro. and Space Sci.* **342** 155 (2012).

[33] S D Sadatian, *Int. J. Theor. Phys.* **53** 675 (2014).

(Received 28 May 2015; accepted 10 June 2015)